Physics Factsheet

April 2001 Number 13

Motion I – Uniform Acceleration

This Factsheet will cover :
- **the basic definitions of speed, velocity and acceleration**
- **the use of the equations of motion for uniform acceleration**
- **the application of the equations of motion to projectile motion**

1. Basic Concepts

Displacement is the distance and direction of a body relative to a given reference point (usually the point at which it starts). As it has both a length and a direction, it is a **vector**. Its SI unit is the **metre** (m). (*eg "200 metres east of my house" is a displacement)*

Velocity is the rate of change of displacement. Accordingly, velocity is also a **vector**. Its SI unit is **metres per second** (ms^{-1}). (*eg 3 ms^{-1} north is a velocity)*

Average velocity = $\dfrac{\textbf{total change in displacement}}{\textbf{time taken}}$

Acceleration is the rate of change of velocity – so, again, is a vector. Its SI unit is **metres per second²** (ms^{-2}).

Average acceleration = $\dfrac{\textbf{total change in velocity}}{\textbf{time taken}}$

Speed and velocity.
Speed is the rate of change of **distance**.

The speed of a body at any instant is equal to the magnitude of its velocity at that instant. However, this is **not** generally the case for average speed and average velocity.

To see why this is, imagine walking 5 m North in 2 s, then 5 m South in 2 s. Since you end up at the point where you started, your overall displacement is zero – so your average **velocity** is **zero**. However, you have travelled a distance of 10 m in 4 s, so your average **speed** is:
10 ÷ 4 = **2.5 ms^{-1}**.

The average speed will **only** be the magnitude of the average velocity if the body concerned is moving in a straight line, without reversing its direction – since then, the distance it moves will always be equal to the magnitude of its displacement.

2. Equations of motion for uniform acceleration

$v = u + at$	u = initial velocity
$v^2 = u^2 + 2as$	v = final velocity
$s = ut + ½at^2$	a = acceleration
$s = ½ (u + v)t$	s = displacement from the starting point
	t = time

It is important to note when using these equations that u, v, a and s can be negative as well as positive.
- Positive and negative values of displacement (s) refer to positions each side of the starting point – for example, if a positive displacement refers to positions above the starting point, then negative ones will refer to positions below it.
- Positive and negative values of velocity (u or v) refer to its direction. For example, if you throw a ball up in the air, then if its initial velocity – when going up – is positive, then its final velocity – when it is coming down – will be negative.
- Any acceleration that has the **opposite** sign to the **velocity** will act as a **retardation** – in other words it will slow the body down. So, if the velocity is positive, a retardation will be negative.

Approach to problems using equations of motion
1. Check that the body is moving with constant acceleration!
2. Write down any of u, v, a, s, t that you know
3. Note down which of u, v, a, s and t that you want (e.g. write a = ?)
4. Decide which equation to use by looking at which of the variables you have got written down in steps 2 and 3. For example, if you have got values for u, t and s, and you want a value for v, then you look for the equation with u, t, s and v in it.
5. Substitute the values you know in, then rearrange.
6. Check that the answer makes sense.

Example 1. A particle is moving in a straight line with constant acceleration. It passes point P with speed 2 ms^{-1}. Ten seconds later, it passes point Q. The distance between P and Q is 40 metres. Find the speed of the particle as it passes point Q.

We know: $u = 2ms^{-1}$ $s = 40$ m $t = 10$ s
We want: $v = ?$

Since we have u, s, t and v involved, use $s = ½ (u + v)t$
Substituting in:

$40 = ½ (2 + v)10$
$40 = 5(2 + v)$
$40 = 10 + 5v$
$30 = 5v$
$v = 6\ ms^{-1}$

Example 2. A particle is moving in a straight line with constant acceleration 0.2 ms^{-2}. After it has moved a total of 20m, its speed is 8ms^{-1}. Find its initial speed.

$a = 0.2\ ms^{-2}$ $s = 20m$ $v = 8\ ms^{-1}$ $u = ?$
So use $v^2 = u^2 + 2as$

$8^2 = u^2 + 2(0.2)(20)$
$64 = u^2 + 8$
$56 = u^2$
$u = 7.48\ ms^{-1}$ (3SF)

Tips:
1. *It is usually easier to put the values in **before** rearranging the equations.*
2. *Take particular care with negative values. On many calculators, if you type in -2^2, you will get the answer –4, rather than the correct value of 4. This is because the calculator squares before "noticing" the minus sign.*

Example 3. A particle starts from rest and moves with constant acceleration 0.5 ms^{-2} in a straight line. Find the time it takes to travel a distance of 8 metres.

u = 0 (as it starts from rest) $a = 0.5\ ms^{-2}$ *s = 8 m t = ?*
Use $s = ut + ½at^2$
$8 = 0t + ½ (0.5)t^2$
$8 = 0.25t^2$
$32 = t^2$
$t = \sqrt{32} = 5.66$ *s (3 SF)*

Vertical motion under gravity
If we assume that air resistance can be ignored, then any body moving under gravity has acceleration *g* downwards, where $g = 9.81\ ms^{-2}$.

Since *g* is constant (for bodies moving close to the earth's surface, this is a good approximation), the equations of motion can be used. The same strategy as before should be used, but in addition, the following should be borne in mind:
- Direction is important. You should always decide which direction you are taking as positive at the start of the problem. You may find it helpful to **always** take **upwards** as **positive**
- The acceleration will always be *g* downwards – or $-g$, if you are taking upwards as positive
- The body will carry on going upwards until **v = 0**
- When the body returns to the ground, **s = 0**

Exam Hint:- *Here are some common mistakes:*
- *Thinking that v = 0 when the body returns to the ground – it isn't!*
- *Thinking that a = 0 when the body is at its highest point – a doesn't change!*
- *Ignoring directions – always ask yourself whether a displacement or velocity is up or down, and so whether it should be put as positive or negative.*
- *Not using the value of g given in the question – you may be told to take it as 9.8, 9.81 or* $10ms^{-2}$ *– and you **must** use the value you are given.*

Example 4. A ball is thrown vertically upwards with speed 20 ms^{-1}. Taking g = 10 ms^{-2}, calculate
a) Its velocity after 1.5 seconds.
b) The height to which it rises
c) The time taken for it to return to the ground.

We will take upwards as positive
a) $u = 20\ ms^{-1}$ $a = -10ms^{-2}$ *t =1.5s v = ?*
So we use v = u + at
v = 20 + (-10)(1.5)
$v = 5ms^{-1}$

b) $u = 20\ ms^{-1}$ *v = 0 (as we want the highest point)* $a = -10ms^{-2}$
s = ?
So we use $v^2 = u^2 + 2as$
$0 = 20^2 + 2(-10)(s)$
0 = 400 – 20s
20s = 400
s = 20 m.

c) $u = 20\ ms^{-1}$ *s = 0 (since it has returned to ground)* $a = -10ms^{-2}$
t = ?

So we use
$s = ut + ½ at^2$
$0 = 20t + ½ (-10)t^2$
$0 = 20t - 5t^2$
0 = 5t(4 – t)
So t = 0 or 4
We want t = 4, since t=0 is when particle was thrown upwards.

NB: If you are unhappy about the factorising of $20t - 5t^2$ used above, then consult Factsheet 15 Maths for Physics: Algebraic Manipulation.

Example 5. A child throws a stone vertically upwards from the top of a cliff with speed 15 ms^{-1}. Five seconds later, it hits the sea below the cliff. Taking g = 9.8 ms^{-2}, calculate
a) the velocity of the stone when it hits the sea
b) the height of the cliff

We will take upwards as positive
a) $u = 15\ ms^{-1}$ $a = -9.8ms^{-2}$ *t = 5 s v = ?*
So use v = u + at
v = 15 + (-9.8)(5)
$v = 15 - 49 = -34ms^{-1}$
So its velocity is 34 ms^{-1} *downwards (because of the minus sign)*

b) $u = 15ms^{-1}$ $a = -9.8ms^{-2}$ *t = 5 s s = ?*
So use $s = ut + ½ at^2$
$s = 15(5) + ½(-9.8)(5)^2$
s = 75 – 122.5 = -47.5 m
So the sea is 47.5 m below the cliff (minus sign – and we'd expect this!)
So the height of the cliff is 47.5 metres

Derivation of the Equations of Motion
Some exam boards require you to derive the equations of motion. If yours does, you must learn the following:

1. We know $a = \frac{\text{change in velocity}}{\text{time}} = \frac{v-u}{t}$
   So, multiplying up: at = v – u
   So, rearranging: u + at = v

2. We know average velocity = $\frac{\text{displacement}}{\text{time}}$
   But average velocity = ½ (u + v)
   So $½ (u + v) = \frac{s}{t}$
   So ½ (u + v)t = s

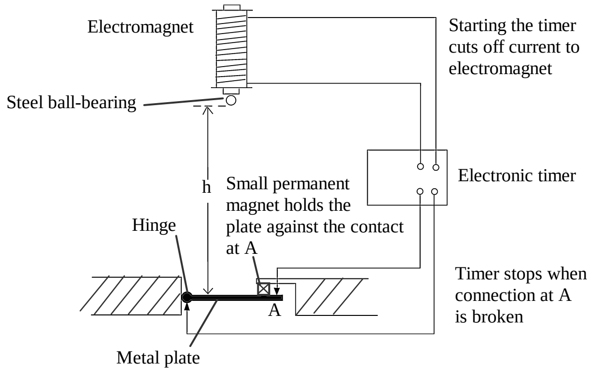
3. From 1, we know v = u + at.
   Substituting this into s = ½ (u + v)t, we get:
   s = ½ (u + u + at)t
   s = ½ (2u + at)t
   s = (u + ½ at)t
   $s = ut + ½ at^2$

4. From 1, by rearranging we know $t = \frac{v-u}{a}$
   Substituting this into s = ½ (u+v)t, we get:
   $$s = ½ (u + v) \frac{(v-u)}{a}$$
   $$s = \frac{(u+v)}{2}\frac{(v-u)}{a} = \frac{(u+v)(v-u)}{2a} = \frac{v^2-u^2}{2a}$$
   $2as = v^2 - u^2$

Typical Exam Question:
A rocket accelerates from rest for 20s with a constant upward acceleration of $10ms^{-2}$. At the end of 20s the fuel is used up and it completes its flight under gravity alone. Assuming that air resistance can be neglected and taking $g = 9.8ms^{-2}$, calculate the:
(a) speed reached after 20s. [2]
(b) height after 20s. [2]
(c) maximum height reached. [3]
(d) speed just before the rocket hits the ground. [2]

Taking upwards as positive:
(a) $u = 0$, $a = 10ms^{-2}$, $t = 20$ s, $v = ?$
$v = u + at$
$v = 0 + 10(20)$ ✓ $= 200ms^{-1}$ ✓
(b) $u = 0$, $a = 10ms^{-2}$, $t = 20$ s, $s = ?$
$s = ut + ½at^2$
$s = 0(20) + ½ (10)(20^2)$ ✓
$s = 2000$ m ✓
(c) *Need to find height reached while moving under gravity. So start from point where fuel runs out.*
$u = 200ms^{-1}$ (from (a)) $a = -9.8\ ms^{-2}$ $v = 0$ $s = ?$
$v^2 = u^2 + 2as$
$0 = 200^2 + 2(-9.8)(s)$ ✓
$s = \frac{200^2}{2 \times 9.8} = 2040m$ (3 SF) ✓
So total height = 2000 + 2040 = 4040m (3 SF) ✓
(d) *Starting from highest point:*
$s = -4040m$ $a = -9.8\ ms^{-2}$ $u = 0$ $v = ?$
$v^2 = u^2 + 2as$
$v^2 = 2(-4040)(-9.8) = 79184$ ✓
$v = 281\ ms^{-1}$ (3 SF) ✓

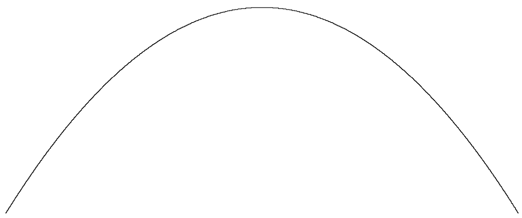
Experimental Determination of *g*
This experiment determines *g* by finding the time a ball-bearing takes to fall from rest through a measured distance. The apparatus is shown below.



Since u = 0, we have $h = ½ gt^2$, so $g = \frac{2h}{t^2}$

To ensure reasonable accuracy,
- the timer must be accurate to 0.01 seconds.
- h is measured from the bottom of the ball-bearing, so the size of the ball-bearing does not introduce errors.
- the experiment should be repeated a number of times and the average found.
- the current in the electromagnet should be reduced to the minimum that will hold the ball-bearing. This reduces the chance of a delay in the ball-bearing being released.

3. Projectiles – motion in two dimensions under gravity
If you throw an object, it follows a **parabolic path** (shown below).



To deal with this situation, we consider the horizontal and vertical components of the motion **seperately**. Again, we assume that air resistance can be neglected
- **horizontally**, there is **no resultant force** on the object, so **its velocity is constant** (and there is **no acceleration**)
- **vertically,** gravity is the only resultant force. So its acceleration is ***g* downwards**

The general approach to problems is very similar to that used in the previous section, but the following should be borne in mind:
- In each part of the question, you must decide whether you need to use the horizontal or the vertical motion
- Again, take care with directions and signs
- At the highest point, v = 0 vertically
- If it returns to the same level at which it started, s = 0 vertically.
- To find its velocity, you need to find the **resultant** of its vertical and horizontal speeds (see Factsheet 02 Vectors & Forces)
- If it is thrown at a speed U and angle α to the horizontal, then
  - the horizontal component of velocity is $U\cos\alpha$
  - the vertical component of velocity is $U\sin\alpha$

  (NB: some exam boards only consider bodies projected horizontally or vertically, rather than at an angle).

Example 1. A vase is thrown out of a first floor window, which is 5 m above the ground, with a horizontal velocity of $4ms^{-1}$.
Taking $g = 10ms^{-2}$, find
a) The time taken for the vase to hit the ground
b) The horizontal distance it travels.
c) Its speed as it hits the ground.

We will take upwards as positive
a) *Since this involves the* ***vertical*** *position of the vase, we must consider the vertical motion*

Vertically: $u = 0$ *(since thrown horizontally)* $a = -10ms^{-2}$
$s = -5m$ *(since it is going downwards)* $t = ?$
Using $s = ut + ½ at^2$:
$-5 = 0(t) + ½ (-10)t^2$
$-5 = -5t^2$
t = 1 second.

b) *We must consider horizontal motion*
$u = 4\ ms^{-1}$ $a = 0$ $t = 1$ s (from a)) $s = ?$

Using $s = ut + ½ at^2$:
$s = 4(1) = 4m$.

c) *This is the magnitude of its velocity.*
We need both horizontal and vertical components.
Vertically: $u = 0$ $a = -10ms^{-2}$ $t = 1s$ (from a))
$v = u + at$
$v = 0 + (-10)(1) = -10ms^{-1}$

Horizontally, velocity is constant, so $v = 4ms^{-1}$

Resultant speed $= \sqrt{(-10)^2 + 4^2} = 10.8\ ms^{-1}$

Example 2. A ball is thrown from ground level with a speed of 20 ms^{-1} at an angle of 30^o to the horizontal. Taking g = 9.8ms^{-2}, find:
a) The greatest height it reaches
b) The time taken for the ball to return to ground level.
c) The horizontal distance the ball travels in this time.

Take upwards as positive:

a) *This is vertical motion.*
$u = 20\sin 30^o\ ms^{-1}$ $a = -9.8\ ms^{-2}$ $v = 0$ $s = ?$

$2as = v^2 - u^2$
$2(-9.8)s = 0 - (20\sin 30^o)^2$
$-19.6s = -100$
$s = 100 \div 19.6 = 5.10\ m$

b) *Again, vertical motion as its level is referred to.*
$u = 20\sin 30^o\ ms^{-1}$ $a = -9.8ms^{-1}$ $s = 0$ $t = ?$

$s = ut + ½ at^2$
$0 = 20\sin 30^o t + ½ (-9.8)t^2$
$0 = 10t - 4.9t^2$
$0 = t(10 - 4.9t)$
$t = 0$ (not applicable) or $10 \div 4.9$
So $t = 2.04$ s (3 SF)

c) *Horizontal motion:*
$u = 20\cos 30^o ms^{-1}$ $a = 0$ $t = 2.04$ s (from b)) $s = ?$

$s = ut + ½ at^2$
$s = 20\cos 30^o (2.04) = 35$ m (2 SF)

Questions

1. Explain the difference between velocity and speed

2. A particle is moving in a straight line with constant acceleration. It passes through point A with speed 4ms^{-1}, and 2 seconds later, through point B with speed 3ms^{-1}. Find
   a) Its acceleration
   b) The distance between A and B

3. A ball is thrown vertically upwards from ground level with speed 25ms^{-1}. Taking g = 10ms^{-2}, find
   a) Its speed when it is 2m above ground level
   b) Its greatest height

4. A ball is thrown horizontally from the top of a cliff with speed 10ms^{-1}, and later falls into the sea. The cliff is 50m high. Taking g = 10ms^{-2}, find
   a) The time taken for the ball to reach the sea
   b) The distance from the bottom of the cliff that it lands
   c) The magnitude and direction of its velocity as it reaches the sea

5. A ball is thrown from ground level with a speed of 28.2 ms^{-1} at an angle of 45^o to the horizontal. Taking g = 9.81ms^{-2}, find
   a) The time it takes to reach its greatest height
   b) The time taken to travel a horizontal distance of 30m

There is a tree 30 m from the point from which the ball was thrown. The ball just passes over the top of it.
   c) Find the height of the tree

Answers
1. See page 1
2. a) a= -0.5 ms^{-2} b) 7 m
3. a) 24 ms^{-1} (2 SF) b) 31 m (2 SF)
4. a) 3.2 s (2 SF) b) 32 m (2 SF)
   c) 34ms^{-1} 73^o below horizontal
5. a) 2.03 s (3 SF) b) 1.50 s (3 SF) c) 18.9 m

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

A dart player throws a dart horizontally. By the time it reaches the dartboard, 3.00m away, it has fallen a height of 0.200m. Taking g as *9.81ms* $^{-2}$, find:

(a) The time of flight [2]
$s = ½ gt^2$
$0.2 = 5t^2$ ✓
$t = 0.2$ s ✗

Examiner's comment: The candidate has used a correct method, but has used g = 10ms^{-2}. Read the question!

(b) The initial velocity [2]
$3 = u0.2$ ✓
$u = 15ms^{-1}$

Examiner's comment: Full marks would have been awarded here as a "follow through" from the error in part a), but for the fact the candidate has not given the direction of the velocity, just its magnitude.

(c) The magnitude and direction of the velocity as it is just about to hit the dartboard. [6]
$v = gt = 2\ ms^{-1}$ ✓✓
$15 + 2 = 17ms^{-1}$ ✗

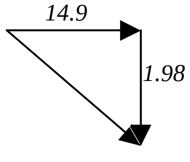
Examiner's comment: The candidate started correctly, by considering the horizontal and vertical components of the velocity. However, the candidate does not seem to appreciate that the two components must be combined as vectors. No attempt has been made to find the direction – the fact that this was asked for should have alerted the candidate to the inadequacy of his/her method.

Examiner's Answers

a) *Vertical:* $s = -0.2$ m $a = -9.81ms^{-2}$ $u = 0$ $t = ?$
$s = ut + ½ at^2$
$-0.2 = -4.905t^2$ ✓
$t = 0.202$ s ✓(3 SF)

b) *Horizontally:* $s = 3m$ $a = 0$ $t = 0.202s$ $u = ?$
$s = ut + ½ at^2$
$3 = u(0.202)$ ✓
$u = 14.9\ ms^{-1}$ (3 SF) ✓

c) *Vertically:* $u = 0$ $t = 0.202$ s $a = -9.81\ ms^{-2}$
$v = u + at$
$v = (-9.81)(0.202)$ ✓$= -1.98\ ms^{-1}$ ✓



magnitude = $\sqrt{(14.9^2 + 1.98^2)}$ ✓= 15 ms^{-1} ✓(2 SF)
at angle $\tan^{-1}(1.98/14.9)$ ✓= 7.6^o (2SF) below horizontal ✓

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Physics Factsheet

April 2003 Number 52

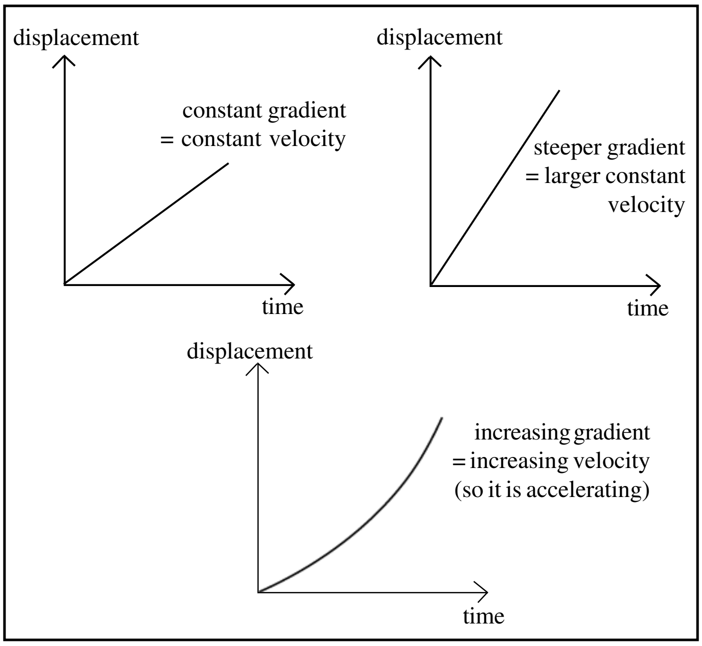
Displacement-time and Velocity-time Graphs

This Factsheet explains how motion can be described using graphs, in particular how displacement-time graphs and velocity-time graphs can be used.

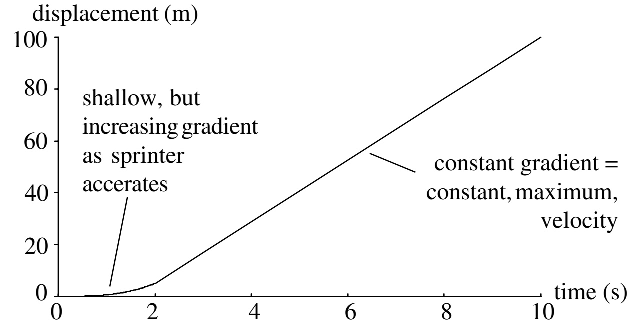
Displacement-time graphs

Displacement, plotted on the vertical axis, represents the straight line distance away from a start point. Time, plotted on the horizontal axis, is the time taken after the start.

- Since velocity = **displacement/time**, the gradient of a displacement-time graph also represents **velocity**. The steeper the gradient the larger the velocity.
- A straight line with a constant gradient will represent an object travelling with constant velocity.
- A curved line with a gradient that changes will represent an object travelling with a varying velocity.

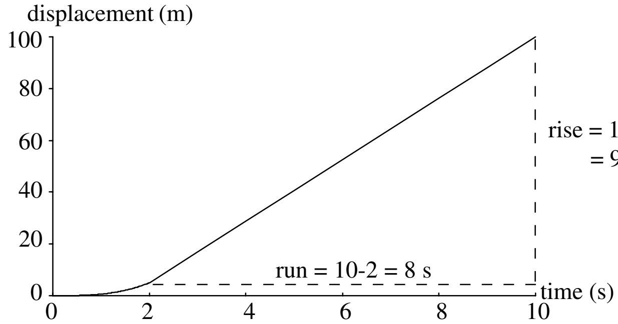


The graph below is a displacement-time for a 100 metre sprinter. The sprinter is slower at the beginning as it takes some time to reach full speed. This is shown by the shallow gradient during the first two seconds of the race, at the start of the graph. As the race progresses the sprinter reaches top speed and is able to maintain this maximum velocity for the rest of the race. This is shown by the gradient of the graph being constant after the first two seconds.



The size of the sprinter's maximum velocity can be obtained from the graph by calculating the gradient of the second section of the graph, beyond the two second point.

The gradient is best calculated by drawing a right angled triangle as shown in the diagram below. The height or 'rise' and length or 'run' of the triangle are then easily read from the graph and used to calculate the velocity.



$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{95}{8} = 11.86 \text{ ms}^{-1}$$

Calculating velocity from a displacement-time graph

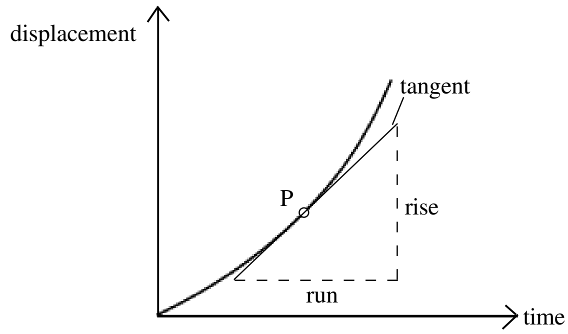
The gradient of a displacement-time graph is equal to velocity.

$$\textit{Velocity} = \textit{gradient} = \frac{\textit{rise}}{\textit{run}}$$

Calculating instantaneous velocities from displacement-time graphs

Calculating the gradient of a graph that does not have a convenient straight line portion requires a tangent to be drawn to the curve.

Consider the displacement-time graph below, which shows a constantly changing gradient indicating that the velocity of the moving object is constantly changing.



The instantaneous velocity of the moving object at point P will be given by the gradient of the curve at this point.

Calculating the gradient of the curve at this point is done by drawing a tangent to the curve. The tangent is the straight line that **just** touches the curve of the graph and has the same gradient as the graph **at this point**. The gradient of the tangent can then be calculated in exactly the same way as described previously, by forming a large right angled triangle and reading the 'rise' and 'run' of the triangle.

Exam Hint: It is a good idea to make the sides of your gradient triangle as long as possible. The reason for this is that a small mistake in a large number is not significant but a small mistake in a small number could easily be. Make sure you draw as long a tangent as you can – in order to make your gradient calculation as accurate as possible.

Look along your tangent, by holding you graph paper up to your eye. You can see how good it is and whether or not it just touches the curve at one point.

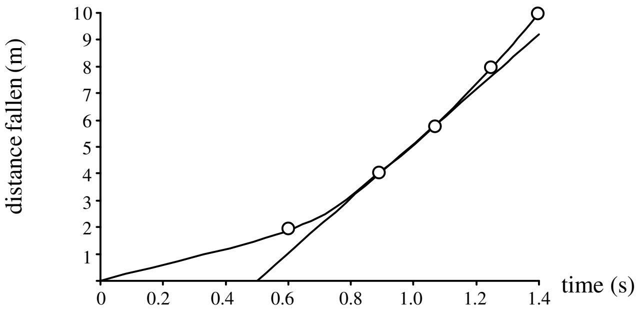
Calculating instantaneous velocities from displacement-time graphs - The instantaneous velocity can be calculated from a curved displacement-time graph by drawing a tangent to the curve at the place where the velocity is required. The gradient of the tangent to the curve will be equal to the instantaneous velocity at that point.

Typical Exam Question
The table of results below were taken for an object being dropped and falling under gravity.

Distance fallen (m)	**0.0**	**2.0**	**4.0**	**6.0**	**8.0**	**10.0**
Time taken (s)	**0.00**	**0.63**	**0.99**	**1.08**	**1.25**	**1.40**

(i) Plot a graph of distance fallen (on the vertical axis) against time taken (on the horizontal axis). [4]
(ii) Explain why your graph is not a straight line [2]
(iii) Calculate the velocity of the object after 1.00 second. [3]

Answer
(i) Graph paper would be supplied with a question like this one. Choose an axis scale that allows the plotted points to fill as much as the graph paper as possible. A mark may be deducted if your points don't fill more than half of the graph paper.



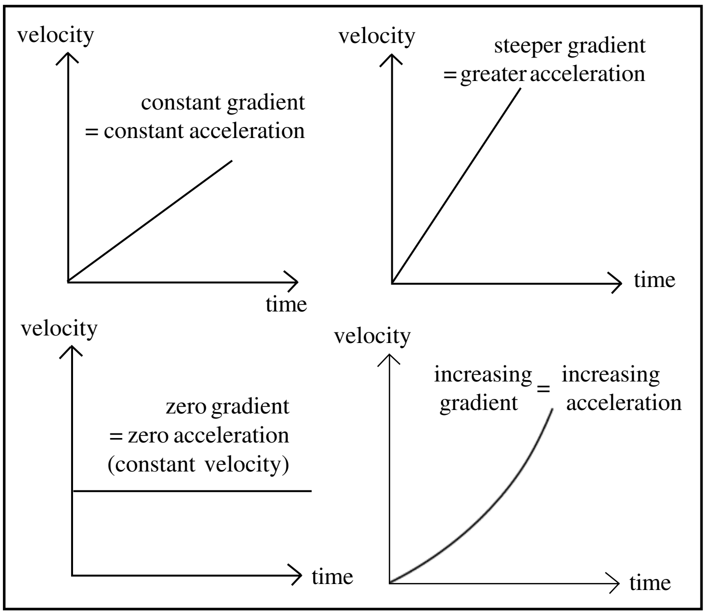
(ii) The increasing gradient of the graph shows an increasing velocity, in other words acceleration. ✓
The acceleration is caused by the gravitational force acting on the object. ✓

(iii) The graph shown as the answer to part (i) has a tangent drawn at a time of 1.00 second.
Velocity = gradient of tangent ✓

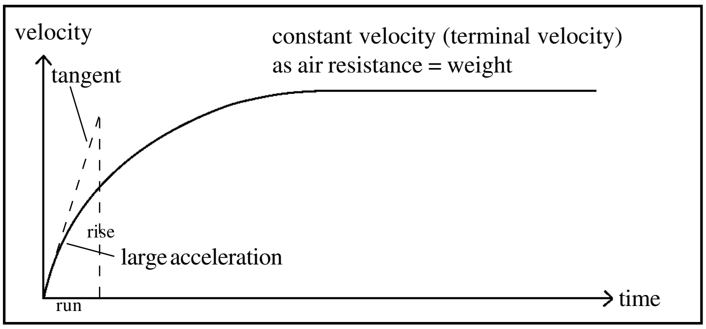
$$= \frac{rise}{run} = \frac{9.2 - 0}{1.4 - 0.5} ✓ = 9.5 \text{ m/s} ✓$$

Velocity-time graphs
Velocity, plotted on the vertical axis, represents the velocity moving away from the start point. The time taken, plotted on the horizontal axis, represents the time taken since the start.

- Since acceleration = $\frac{\text{change in velocity}}{\text{time}}$
  the gradient of a velocity-time graph also represents acceleration.
- The steeper the gradient the larger the acceleration
- A straight line with a constant gradient will represent an object travelling with constant acceleration.
- A curved line with a gradient that changes will represent an object travelling with a varying acceleration.



The graph below represents the velocity-time graph for a freefalling skydiver.



The gradient is initially large as the skydiver is accelerating with the acceleration due to gravity.

The gradient of the graph gradually decreases showing the acceleration of the skydiver to be decreasing as the air resistance on the skydiver increases. Eventually the air resistance on the skydiver is equal to his weight; there is no resultant force so there is no acceleration. The skydiver falls at constant velocity, shown by the horizontal line on the graph; zero gradient implies zero acceleration and constant velocity.

The size of the initial acceleration of the skydiver can be determined by calculating the initial gradient of the graph. This is done in exactly the same way as for any other graph, by taking a rise and run from the graph, as we looked at with displacement – time graphs, using a right angled triangle drawn on the graph.

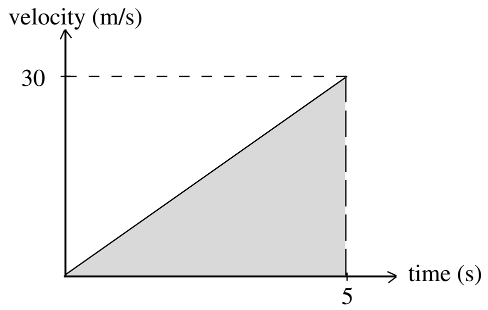
Calculating acceleration from a velocity-time graph
The gradient of a velocity-time graph is equal to acceleration. The height or 'rise' and length or 'run' of a part of the graph is measured.

$$Acceleration = \frac{\text{change in velocity}}{\text{time}} = gradient = \frac{rise}{run}$$

The total displacement during a journey can also be calculated from a velocity-time graph. The area beneath the line on a velocity-time graph gives the total displacement.

The graph below is a velocity-time graph for an accelerating car. The graph is a straight line showing that the car has constant acceleration. The displacement of the accelerating car after this 5 second journey can be determined by calculating the area beneath the graph.

The line of the graph forms a triangle with the horizontal axis so the area of the triangle can be calculated.



Total displacement = area beneath graph = area of triangle shape
$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 5 \times 30 = 75\text{m}$

> ***Calculating the total displacement from a velocity-time graph***
> *The total displacement is equal to the area beneath the line on a velocity-time graph for the time considered.*

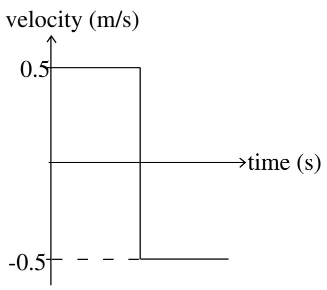
> ***Exam Hint:*** *The majority of velocity-time graphs that will be used in an exam will consist of sections of constant acceleration or constant velocity. This means that the graph can be split into a series of triangles and rectangles when calculating the area beneath the graph.*

Velocity as a vector

Velocity is a vector quantity. This means that velocities are described by two things; the size or magnitude of the velocity and the direction of the velocity.

- The size of the velocity is simply described by a number with a unit in the usual way, e.g a cyclist moving at 5 ms^{-1}.
- The direction of the velocity in journeys that can only go back and forth in a straight line is described by adding a sign to the size of the velocity.
- A positive sign would mean travelling in one direction and a negative sign would mean travelling along the same line but in the opposite direction. Therefore, a swimmer who is swimming lengths of the pool, there and back, would have a positive velocity while swimming to the far end of the pool but a negative velocity while swimming back to the start.
- Velocity-time graphs can also show negative velocities by having negative values plotted on the vertical axis.

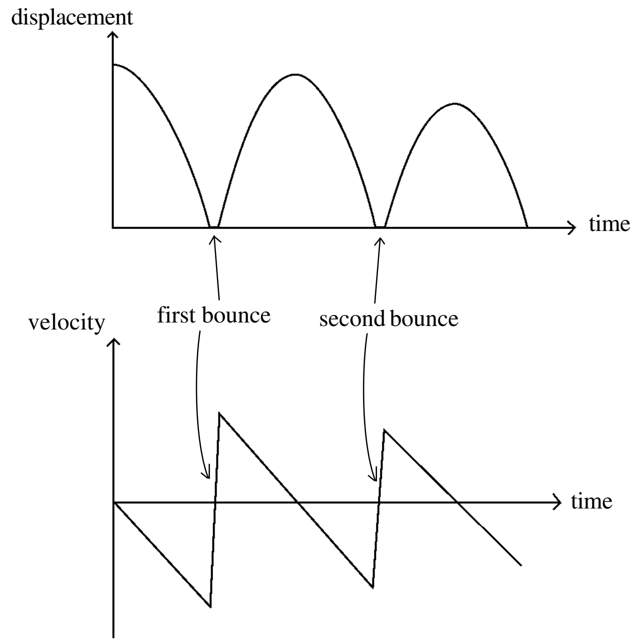
The swimmer, swimming at a constant velocity of 0.50 ms^{-1}, would have a velocity-time graph as shown below. The velocity switches from positive to negative as the swimmer turns around and starts to swim in the opposite direction along the pool.



> ***Velocity is a vector***
> *Velocity is a vector. A vector is a measured quantity that is described by a magnitude, (or size), and a direction.*
> *For motion along a straight line this means that moving in one direction along the line will be a positive velocity and moving in the opposite direction will be called a negative velocity.*

Displacement-time graph and velocity-time graph for a bouncing ball

The displacement-time and velocity-time graphs for a bouncing ball are specifically mentioned in several A-level specifications. The two graphs below are for a ball that is initially dropped from someone's hand and allowed to bounce on the floor.



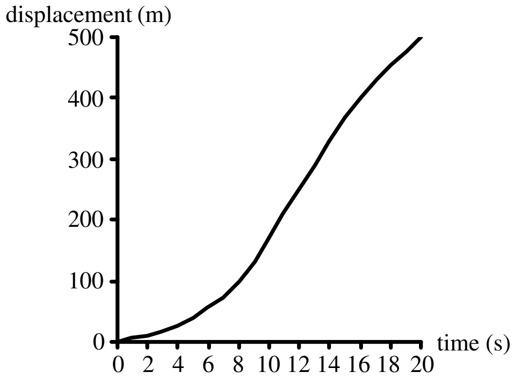
Displacement-time graph
- Zero displacement is defined as the floor.
- The gradient of the displacement-time graph is velocity. The gradient of the graph is negative and becomes increasingly large as the ball falls and speeds up.
- When the ball hits the ground, it bounces back up and the gradient becomes positive.
- The gradient then decreases until the ball is at the top of its path.
- The ball then drops downwards once more.

Velocity-time graph
- The ball is dropped from rest and so the initial velocity is zero.
- Velocity downwards has been given a negative sign and so the velocity then becomes a bigger negative number as the ball accelerates downwards.
- The gradient of the graph is acceleration and this is constant at $-9.81\ ms^{-2}$ as this is acceleration due to gravity.
- When the ball bounces it rapidly comes to a stop before bouncing back, upwards, with a positive velocity.
- The ball will then slow down until, at the top of its path, it will instantaneously have zero velocity before heading back towards the ground.

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

The graph below represents the displacement of a drag racing car along a straight track.



(a) (i) Calculate the instantaneous velocity of the car 12 seconds after the start. [2]

$$\text{velocity} = \frac{\text{displacement}}{\text{time}} = \frac{250}{12} = 21\ ms^{-1}$$

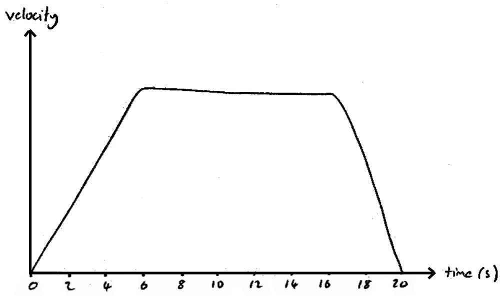
The student has simply substituted values of displacement and time from the point at 12 seconds on the graph. No attempt has been made to determine the gradient of this straight line portion of the graph.

(ii) Calculate the velocity of the car 6.0 seconds after the start. [3]

$$\text{velocity} = \frac{\text{displacement}}{\text{time}} = \frac{50}{6} = 8.3\ ms^{-1}$$

Again, the values from the point at 6 seconds have been substituted. The graph is a curve at this point and a tangent should be drawn on the graph in order to calculate an instantaneous gradient.

(b) On the axes below sketch a velocity-time graph for the car over the same period of time. [4]



Even though the question says 'sketch' values should be placed on the vertical axis as we have just calculated 2 points from the first part of the question.

(c) Without any calculation state what the area beneath your velocity-time graph represents and what the value should be. [2]

Area beneath the graph represents length of race and it should be 500m

The candidate would be awarded both marks for this part of the question but more detail could have been given for the first part of the answer by mentioning the total displacement of the car from the start position.

Examiner's Answer

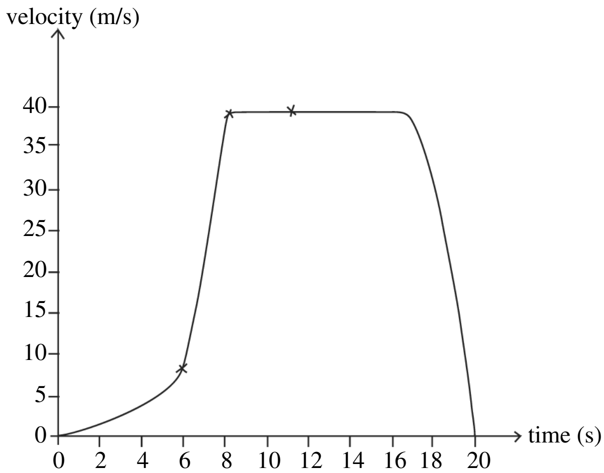
(a) (i) $\text{velocity} = \text{gradient of graph} = \frac{rise}{run} = \frac{400-100}{16-8} = 37.5\ ms^{-1}$ ✓✓

(ii) Instantaneous velocity = gradient of tangent

$= \frac{rise}{run} = \frac{115-0}{10-2} = 14.4\ ms^{-1}$ ✓✓

Please note that actual numbers for rise and run will vary depending on the size of the line used to calculate the gradient but the final answers should all be very similar.

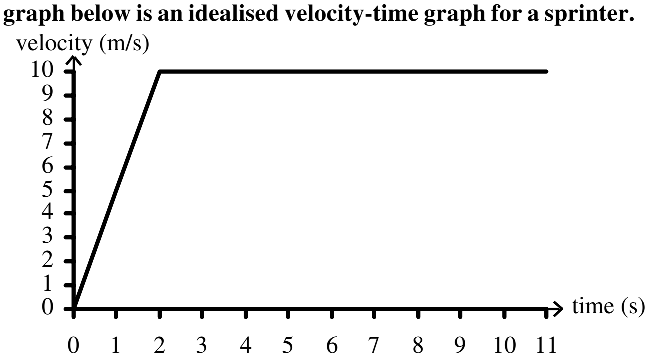
(b)



(c) The area beneath the graph represents the total displacement of the car, which is the distance the car has travelled along the straight track. ✓
The area beneath the graph should be the total displacement given on the displacement-time graph = 500m. ✓

Typical Exam Question

The graph below is an idealised velocity-time graph for a sprinter.



(a) What is the initial acceleration of the sprinter? [3]

(b) Over what distance did the sprinter race? [3]

(c) What was the *average* velocity of the sprinter for the entire race? [2]

Answer

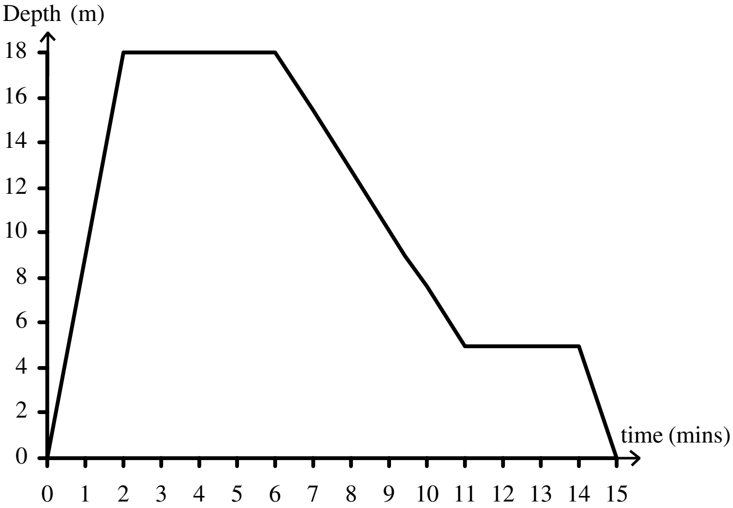
(a) $\text{acceleration} = \frac{\text{change in velocity}}{\text{time}} = \text{gradient} = \frac{rise}{run} = \frac{10}{2} = 5\ ms^{-2}$ ✓✓✓

(b) The displacement of the sprinter will give the length of the race.
The area beneath the graph gives the displacement. ✓
The graph can be split up into a triangle for the first 2 seconds and a rectangle for the final 9 seconds.
The total displacement will be given by the sum of the two areas.
Displacement = total area beneath the graph ✓
$= (\tfrac{1}{2} \times 2 \times 10) + (9 \times 10) = 100\ m$ ✓

(c) $\text{average velocity} = \frac{\text{total displacement}}{\text{total time taken}} = \frac{100}{11} = 9.1 ms^{-1}$ ✓✓

Qualitative (Concept) Test
1. What does the gradient of a displacement-time graph represent?
2. How would the gradient of a curved graph be calculated?
3. What does the gradient of a velocity-time graph represent?
4. What does the area beneath a velocity-time graph represent?
5. What is a vector and how is the vector nature of velocity in a straight line shown?
6. Sketch the displacement-time and velocity-time graph of a bouncing ball and label the important features of both.

Quantitative (Calculation) Test
1. The graph below represents the depth of a scuba diver during a 15 minute dive.



(a) During which period of the dive was the diver ascending the quickest? [1]
(b) How long did the diver stay at the bottom of the sea, a depth of 18m? [1]
(c) What was the vertical velocity of the diver during his descent? [3]

2. The table of results below were taken for a cyclist travelling along a straight road.

Velocity (ms^{-1})	0	5	10	15	15	12	9	6	3	0
Time taken (s)	0	10	20	30	40	50	60	70	80	90

(a) Draw a graph of velocity on the vertical axis against time on the horizontal axis for the journey. [4]
(b) What is the initial acceleration of the cyclist? [2]
(c) Calculate the deceleration of the cyclist in the final 50 seconds of the journey. [3]
(d) Calculate the total distance that the cyclist travelled along the straight road. [3]
(e) Calculate the average velocity of the cyclist for the entire journey. [2]

Quantitative Test Answers
1. (a) In the final minute of the dive.
(b) 4 minutes
(c) $\text{velocity} = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{18}{2 \times 60} = 0.15\ ms^{-1}$

2. (b) $\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$

$= \frac{(15-0)}{(30-0)} = 0.50\ ms^{-2}$

(c) $\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} = \text{gradient} = \frac{\text{rise}}{\text{run}}$

$= \frac{(0-15)}{(90-40)} = -0.30\ ms^{-2}$

deceleration = $0.30\ ms^{-2}$

(d) Total distance = area beneath graph
$= (½ \times 30 \times 15) + (10 \times 15) + (½ \times 50 \times 15) = 750m$

(e) $\text{average velocity} = \frac{\text{total distance}}{\text{time}} = \frac{750}{90} = 8.3\ ms^{-1}$

Acknowledgements: *This Physics Factsheet was researched and written by* ***Jason Slack.***
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Interpreting and Drawing Graphs of Motion

In Factsheet 116, we looked at graphical work with electricity. This was in response to Examiner's Reports suggesting that students have displayed weaknesses in sketching and interpreting graphs.

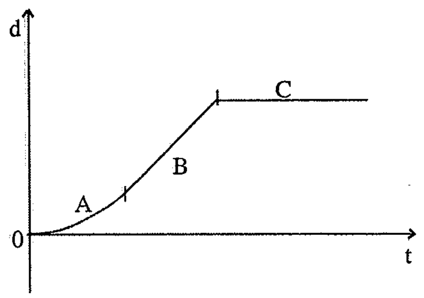
In this Factsheet we will follow this up with similar work within Motion – a topic that lends itself to graphical work. We will be looking at selected examples within this topic.

Velocity-time and displacement-time graphs

We usually use the terms **velocity** and **displacement** here because they are vector quantities (unlike **speed** and **distance**), and so can have a direction rather than just a magnitude.

Generally we look at linear motion, and positive and negative values represent the forward and backward directions.

Displacement-time (d-t)



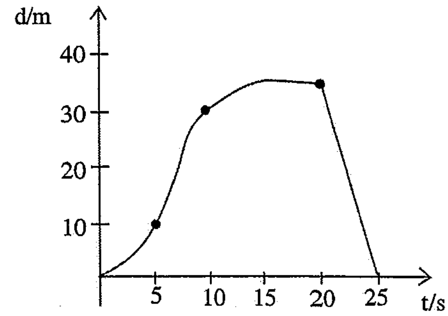
This graph is divided into three sections.

- In **A**, the rate of change of displacement with time is increasing. This implies acceleration.
- In **B**, the displacement increases at a steady rate. This implies uniform speed (or uniform velocity, more correctly).
- In **C**, there is no change in displacement. The object is stationary.

> ***Example: In A, do we have uniform acceleration?***
>
> ***Answer:*** *There is not enough information to tell. But the fact that there is a smooth transition to uniform speed suggests that the acceleration is slowly decreasing to zero at the end of section A.*

> ***Exam Hint:*** *Don't attempt to draw too many conclusions from sketch graphs. Without numerical data being provided, there is often not enough information to be certain about anything except trends.*

Considerably more detail is available in this graph:



(a) What is the average velocity for the journey?
Zero, as you end up back at the starting point (velocity is a vector)

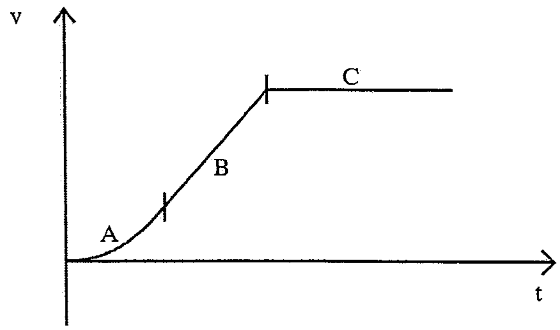
(b) What is the average speed for the journey?
$V_{ave} = 70/25 = 2.8ms^{-1}$

(c) What is the average speed between the 5 and 10 second marks?
$V_{ave} = 20/5 = 4.0ms^{-1}$

(d) What can we learn from the area under the graph?
Nothing of real value (displacement × time?)

(e) What would a negative value for displacement tell you?
That you were now behind your original starting position.

Velocity-time (v-t)

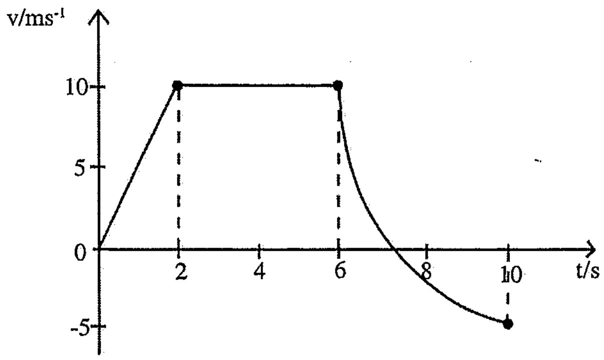


On first glance, this graph looks like the earlier one. But this is a velocity-time graph, not a displacement-time graph.

- In **A,** the rate of change of velocity with time is increasing. This shows **increasing** acceleration.
- In **B,** the velocity increases at a steady rate. This acceleration is **uniform**.
- In **C**, there is uniform velocity (as opposed to the object being stationary in the d-t graph).

> ***Exam Hint:*** *An identical looking graph describes totally different motion. Do not confuse velocity-time and displacement-time graphs.*

Example: This v-t graph represents the motion of an object:

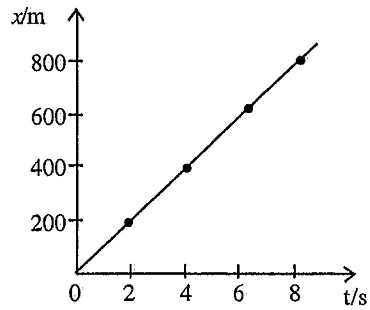


(a) Find the acceleration in the first 2 seconds.
(b) Find the distance travelled in the first 6 seconds.
(c) Estimate the **distance** travelled over the 10 seconds.
(d) Estimate the **displacement** over the 10 seconds.

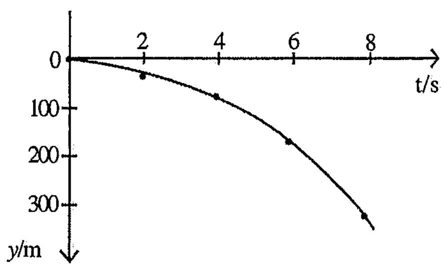
Answer
(a) $a = 10/2 = 5ms^{-2}$
(b) Distance = area under graph = 10 + 40 = 50m
(c) Estimate areas between 6 and 10 seconds.
Distance = 10 + 40 + 5 + 7 = 62m
(d) Subtract motion back towards start (negative velocity)
Displacement = 10 + 40 + 5 - 7 = 48m forwards.

Projectile motion graphs
A projectile, e.g. cannonball, travels in both the **x** and **y** planes. Its position also changes with time. We cannot put all three variables on one graph. But obtaining two graphs using our knowledge of physics, allows us to predict the third graph.

Let's look at a cannonball fired horizontally at $100ms^{-1}$ from a cliff.
(a) **x-direction**
Ignoring air resistance, it should travel at constant speed.



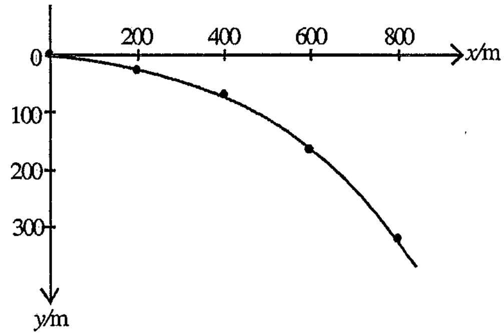
(b) **y-direction**
The Earth's gravitational field will cause an acceleration of approximately $10ms^{-2}$ vertically downwards.
After 2 seconds, $y = 0.5 \times 10 \times 2^2 = 20m$
After 4 seconds, $y = 0.5 \times 10 \times 4^2 = 80m$
After 6 seconds, y = 180m
After 8 seconds, y = 320m



We can now make a table of **x** and **y** at equal times:

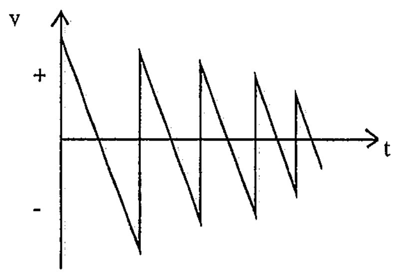
t/s	x/m	y/m
0	0	0
2	200	20
4	400	80
6	600	180
8	800	320

And then draw a graph of the cannonball's path:



It is often possible to use two relationships to determine a third. We generally see this in algebra, but it can be performed graphically as well.

We often see velocity-time graphs of a ball bouncing on a hard surface:



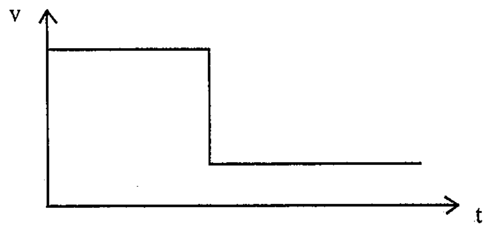
This graph displays a number of Physics ideas:
(a) Why is the downwards gradient always the same?
The rate of change of velocity is the acceleration due to the earth's gravitational field. This is a constant, of course ($9.81ms^{-2}$).
(b) Why is the upwards gradient almost vertical each time?
The sudden change of velocity occurs as the ball distorts, then bounces. This happens very quickly.
(c) Why does the amplitude decrease from bounce to bounce?
Kinetic energy is lost to heat energy as the ball distorts and bounces.
(d) Why is each positive peak followed by a negative peak of exactly the same displacement?
Ignoring air resistance, there should be no loss of energy as the ball rises, then falls, between bounces.
(e) How could you find the height reached each time?
For a v-t graph the distance travelled up or down would be the area under each positive or negative triangle.

Exam Hint: *Presented with graphs of this type, be prepared to perform calculations concerning acceleration, maximum height, kinetic energy (and gravitational potential energy), percentage of energy lost in each bounce, etc. (See the questions at the end of the Factsheet.)*

Collisions

Momentum is conserved in collisions, but some kinetic energy is usually lost to heat and sound. As a result, we usually concentrate on conservation of momentum with collisions.

Suppose a moving trolley **A** collides with a stationary trolley **B**, and the pair move off together. Here is a possible graph for trolley **A**:

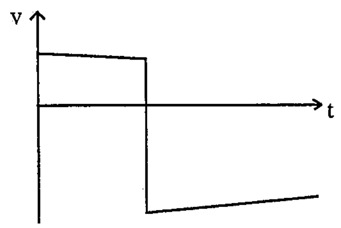


What could we deduce from this simple graph?

(a) From conservation of momentum and the large drop in speed for **A**, we could show that trolley **B** has a larger mass than trolley **A**.

(b) From the horizontal lines (uniform speed), we can see that there is negligible friction or air resistance acting.

Suppose both trolleys were moving before the collision, and the graph for trolley **A** resembles this:



We could deduce:

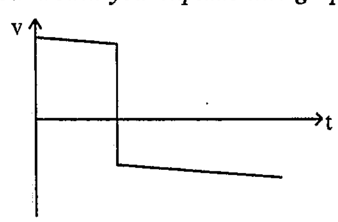
(a) **B** was moving towards **A** before the collision.

(b) As the final velocity of **A** is in the opposite direction, **B** was either moving faster than **A**, or **B** has a larger mass, or both.

(c) There is significant friction or air resistance. (The velocity is decreasing as time passes).

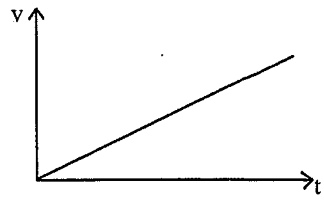
> ***Example: Can you use Conservation of Momentum ideas when friction is acting on the trolleys?***
>
> ***Answer:*** *Yes. The change in momentum is almost instantaneous as the collision occurs. Friction should have little effect over this very tiny time interval.*

> ***Exam Hint:*** *When friction is changing the speed of the trolleys before and after a collision, you must use the velocities **immediately** before and after the collision in any Conservation of Momentum calculations.*

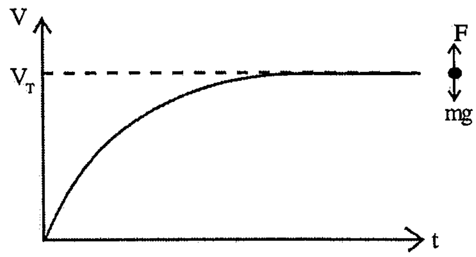
> ***Example: How would you explain this graph for trolley A?***
>
> 
>
>
> ***Answer:*** *This is similar to the previous collision, except that A* <u>*accelerates*</u> *(in the reverse direction) after the collision. The obvious explanation is that it is now travelling downhill (perhaps on a ramp), or with a strong tailwind.*

Objects falling under gravity

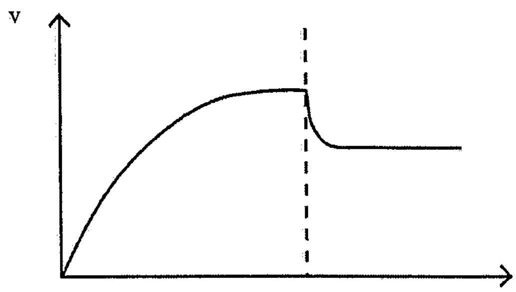
The motion of a skydiver in freefall produces a simple constant acceleration graph at the beginning of her descent.



But as air friction increases with speed:



The terminal velocity v_T is reached when air resistance increases to balance her weight.

> ***Example: Sketch a v-t graph for a body falling into the sea, after reaching terminal velocity in air.***
>
> ***Answer***
>
> 
>
>
> *A second terminal velocity is quickly achieved, when the water resistance and upthrust balance her weight. In which section of the graph are the resistive forces greater than her weight?*

> ***Exam Hint:*** *Be prepared to show the forces acting, and their relative magnitudes, at various points during the object's descent. (A question at the end illustrates this.)*

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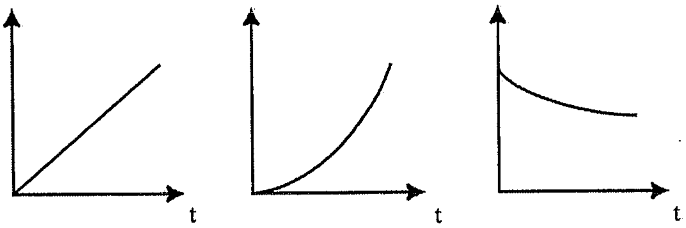
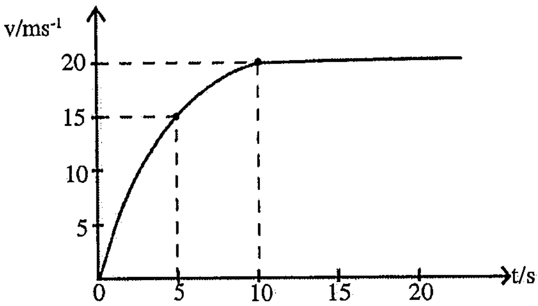
Practice Questions

1. Identify each graph as a velocity-time (v-t) graph or as a displacement-time (d-t) graph:

(a) uniform velocity (b) uniform acceleration (c) projectile with air resistance



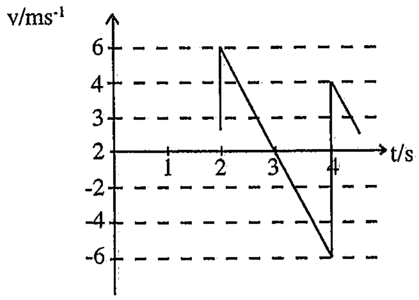
2. This is a graph of a skydiver in the first 20s of her fall.



Find:
(a) the acceleration at the 15s mark
(b) the average acceleration over the first 5 seconds
(c) an estimate of the distance fallen during the 20 seconds
(c) the approximate average speed of descent.

3. For this same skydiver, draw vector diagrams for the forces acting on her after zero, five, and ten seconds.

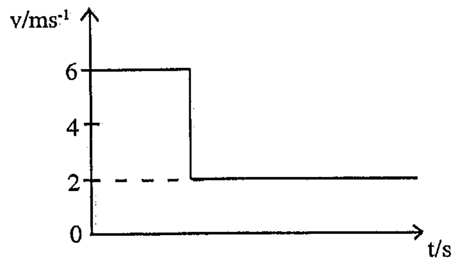
4. This is a section of a v-t graph for a bouncing ball of mass 200g (not necessarily on the Earth).



Find:
(a) the KE of the ball after the first bounce shown
(b) the KE of the ball after the second bounce
(c) the percentage energy lost in this bounce
(d) the height reached after the first bounce
(e) the GPE of the ball at the 3 second mark.

5. (a) Sketch the x-y graph for a projectile fired at an angle of about 30deg above the horizontal. Ignore air resistance.
(b) Sketch the same graph if there was a significant amount of air resistance.
(c) Explain two effects of the air resistance on the path of the projectile.

6. A 10kg trolley collides and sticks to a stationary trolley. This is the graph of the 10kg trolley's motion. Find the mass of the other trolley.



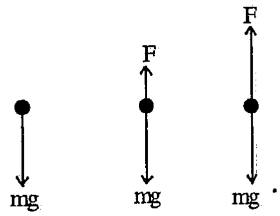
7. For a bouncing ball, sketch a graph of KE against time. Explain your graph.

Answers

1. (a) d-t (b) d-t (c) v-t

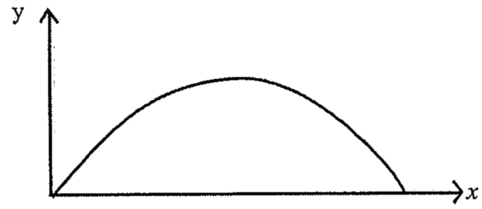
2. (a) zero
(b) $3ms^{-2}$
(c) Approximately 130 + 200 = 330m (adding the areas)
(d) $v = d/t = 330 / 20 = 16.5ms^{-1}$

3.

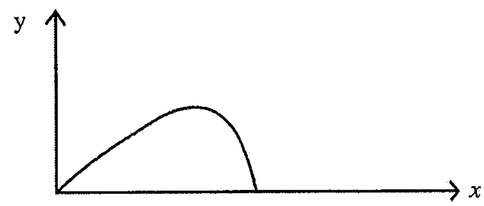


4. (a) $KE = ½ mv^2 = ½ \times 0.2 \times 36 = 3.6J$
(b) 1.6J
(c) % energy lost = $(2.0/36) \times 100 = 56\%$
(d) Height = area = $½ \times 1.0 \times 6 = 3m$
(e) v = 0, so all KE has become GPE. So GPE = 3.6J

5. (a)



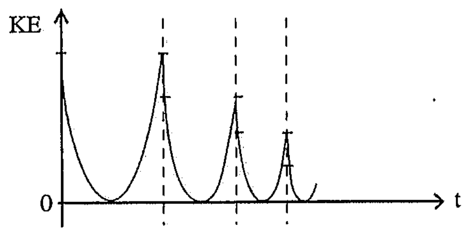
(b)



(c) Slower speed at every point, maximum height less, range less, non-symmetrical path, etc.

6. Conservation of momentum
$10 \times 6 = (10 + m) \times 2 \quad m = 20kg$

7.



Curve as $KE \propto v^2$
KE positive even when v negative, as $KE \propto v^2$
KE lost during each contact with floor.

Physics Factsheet

www.curriculumpress.co.uk Number 63

Solving problems on projectiles

This Factsheet looks at the motion of objects moving under gravity. They are not being driven forward but may have been given a start velocity in any direction. This applies to many sports like golf, tennis, discus, ski jump, target shooting, long jump and stunt riding. The same approach also applies to charged particles moving in a uniform electric field. Any air resistance is ignored in these questions.

- You need to know the standard equations of motion (Factsheet 13); the ones used most commonly here will be $v = u + at$, $2as = v^2 - u^2$ and $s = ut + ½at^2$
- The **up** direction is taken as **positive** for displacement, velocity and acceleration - hence acceleration due to gravity, g, is negative.
- You should also be familiar with vector addition and resolving into components (Factsheet 2)

General Points

The strategy is to consider the horizontal and vertical components of the motion separately, and use the equations of motion in each direction.

- The vertical acceleration is always -9.8 ms^{-2} (due to gravity)
- The projectile will continue to move upwards until the vertical component of its velocity is zero
- The horizontal acceleration is always zero
- The horizontal velocity is constant

Strategies

There are various quantities that you are commonly asked to find. These are the strategies to find them:

- Time to the highest point — use $v = u + at$ vertically, with $v = 0$
- Greatest vertical height — use $2as = v^2 - u^2$ vertically, with $v = 0$, or use $s = ut + ½at^2$ if the time is known
- Time taken for it to reach a particular height — use $s = ut + ½at^2$ vertically (if it is returning to the same height at which it started - eg returning to the ground - take $s = 0$)
- Total horizontal distance travelled — find the time taken for it to finish its journey (as above), then use $s = ut$ horizontally

Types of question

There are three common situations, when an object is:

1. Thrown vertically upwards or downwards 2. Projected with a horizontal velocity 3. Projected at an angle to the horizontal

The first one is not conventionally called a projectile, but can be examined, and understanding it helps with the last two.

1. Object thrown vertically

- No horizontal component - consider **vertical motion only**.
- Start velocity: u has +ve value if thrown upwards, –ve value if downwards
- There is vertical acceleration g = - 9.8 ms^{-2}.
- If the object is thrown upwards, it continues to rise until $v = 0$

For an object thrown vertically

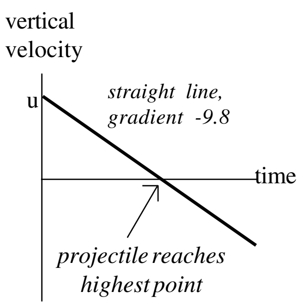
$v = u - 9.8t$ — *u = initial vertical velocity*

$s = ut - 4.9t^2$ — *v = velocity vertically upward*

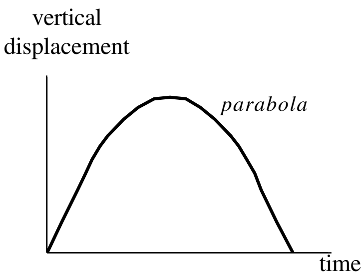
s = displacement vertically upward

If the particle is thrown **upwards**, the graphs are as follows:

velocity-time graph



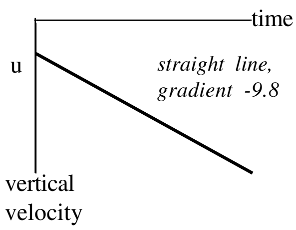
displacement-time graph



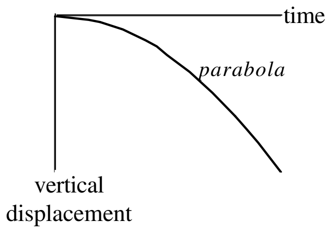
Note: This assumes the projectile returns to the same vertical level from which it started.

If the particle is thrown **downwards**, the graphs are as follows:

velocity-time graph



displacement-time graph



Worked example: A ball is thrown straight up with a speed of 23 ms^{-1}. Find the time taken to reach the top of flight and the height gained.

Here we have $v = 23 - 9.8\,t$

So at the top of the flight, $t = 23/9.8 = 2.35$ *seconds*

Since we have found the time, use it to find the height:

$s = 23t - ½\,(9.8)t^2$

$= 23(23/9.8) - ½\,(9.8)(23/9.8)^2$

$= 27.0\ ms^{-1}$

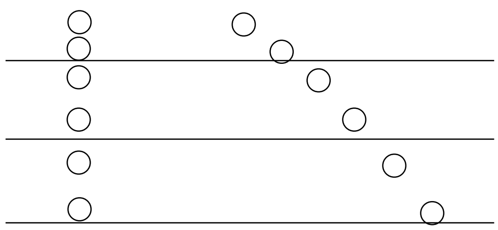
Note that if you were unsure about the accuracy of the value calculated for the time, it would be possible to calculate the height without it, using the equation $2as = v^2 - u^2$

Exam Hint *Note how the original, rather than rounded, time has been used in the second part of the calculation to avoid rounding errors.*

2. Object projected horizontally

- Start velocity has no **vertical** component, only a horizontal one
- Acceleration is $g = -9.8\ ms^{-2}$ vertically; no horizontal acceleration
- Object continues to move until it falls to the ground.

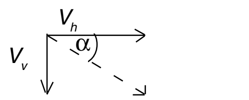
The horizontal velocity is **not altered by acceleration downwards**.



Neither is the acceleration downwards changed by the horizontal motion. **They are independent**.

*All objects launched horizontally or dropped from the **same height** will hit the ground **at the same time**. Their vertical component of velocity is the same at all times.*

Exam Hint: *If the question asks for a velocity at a certain time or point with no direction mentioned then it will mean **resultant velocity.** You should give both magnitude and direction (angle) in your answer.*

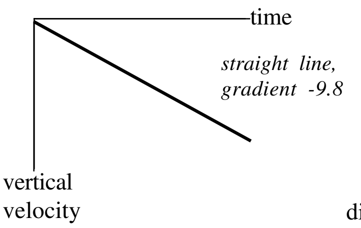


For an object projected horizontally

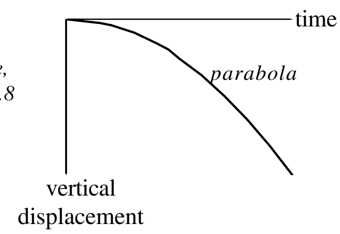
$v_v = -9.8t$	$v_h = u_h$	u_h = start horizontal velocity
$s_v = -4.9t^2$	$s_h = u_h t$	v_v, v_h = velocity components
$2(-9.8)s_v = v_v^2$		s_v, s_h = displacement components

Vertical Motion

velocity-time graph

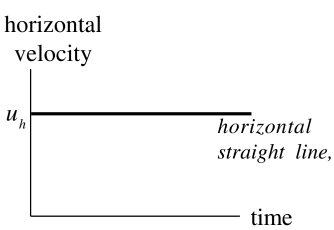


displacement-time graph

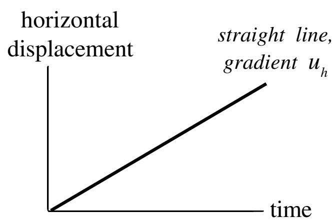


Horizontal Motion

velocity-time graph



displacement-time graph



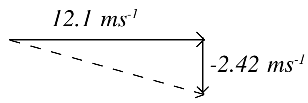
Typical Exam Question

A dartboard is 3.0 m away. A dart is thrown horizontally from height 1.90 m and hits the board at 1.60 m. Calculate the
(a) time of flight
(b) initial speed of the dart
(c) speed of the dart when it hits the board.

(a) *whatever the horizontal speed, it takes the same time to fall 0.30 m.*
from $h = \frac{1}{2} g t^2$ *(1mark),* $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 0.30}{9.8}} = 0.247s$ *(1 mark)*

(b) *During this time it travels the 3.0 m (1 mark).*
so the speed $= \frac{distance}{time} = \frac{3.0}{0.247} = 12.1\ ms^{-1}$ *(1 mark)*

(c) *Note the question asks for **speed** so you don't need the angle but you need the magnitude of the resultant velocity.*
the vertical velocity $v = at = -9.8 \times 0.247 = -2.42\ ms^{-1}$ *downwards (1 mark)*
combine this with the horizontal velocity using Pythagoras' theorem



magnitude of resultant velocity $= \sqrt{12.1^2 + (-2.42)^2} = 12.34\ ms^{-1}$ *(1 mark)*

(the minus doesn't affect the answer because it is squared but it is consistent to put it in)

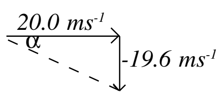
Typical Exam Question

A boy throws a stone horizontally off a cliff. It hits the sea 2 seconds later, at a distance of 40 *m* from the foot of the cliff.
Calculate the
(a) height of the cliff
(b) initial speed of the stone
(c) direction in which the stone is moving when it strikes the water.

(a) Consider vertical motion: $s_v = -4.9t^2$
$s = -4.9(2^2) = -19.6m$
Cliff is 19.6m high

(b) Consider horizontal motion: $s = ut$
$40 = u2$
$u = 20ms^{-1}$

(c) Direction it is moving in is direction of velocity
$v_h = 20ms^{-1}$ $\quad v_v = -9.8(2) = -19.6ms^{-1}$



Angle $= \tan^{-1}(19.6/20)$
$= 44^o$

So direction of motion is 44° below the horizontal

Typical Exam Question

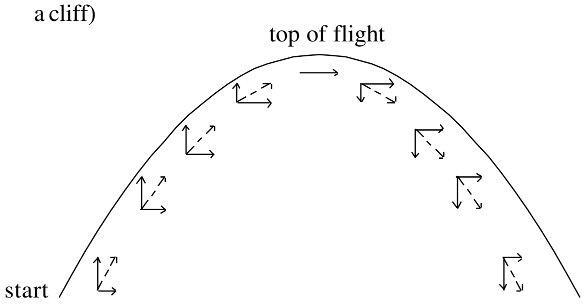
A bullet is fired horizontally at 850ms⁻¹ from a height of 1.8 m above the ground. Calculate the
(a) time it takes to hit the ground
(b) horizontal distance it travels

(a) Consider vertical motion: $s = -4.9t^2$
$-1.8 = -4.9t^2$
$t = 0.61$ *seconds*

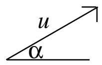
(b) Consider horizontal motion: $s = 850t$
$s = 950(0.60609) = 580m$ *(2SF)*

3. Launched at an angle to the horizontal

- Start velocity has both horizontal and vertical components
- Acceleration is $g = -9.8$ ms^{-2} vertically; no horizontal acceleration
- Object continues to move until it falls to the ground.
- In many cases, the object returns to the same height form which it started, but this is not always the case (eg object thrown upwards from a cliff)



The components of the start velocity are given by



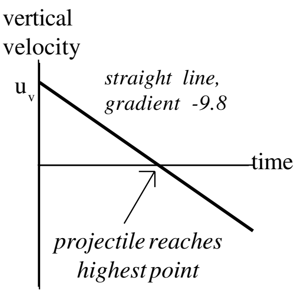
$u_h = u\cos\alpha$
$u_v = u\sin\alpha$

For a general projectile

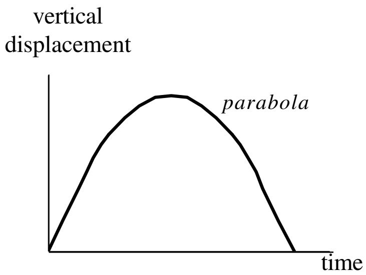
$v_v = u_v - 9.8t$	$v_h = u_h$	u_v, u_h = start velocity components
$s_v = u_v t - 4.9t^2$	$s_h = u_h t$	v_v, v_h = velocity components
$2(-9.8)s_v = v_v^2 - u_v^2$		s_v, s_h = displacement components

Vertical Motion

velocity-time graph

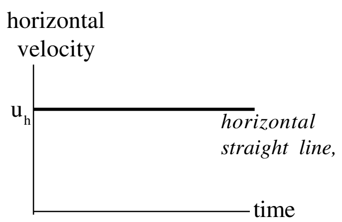


displacement-time graph

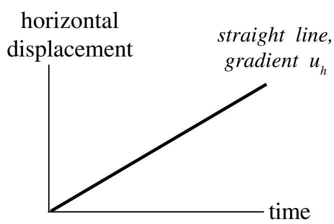


Horizontal Motion

velocity-time graph



displacement-time graph



Example (long jump)

If the maximum running speed is about 11 ms^{-1} and you could take off at 45^o with this speed, find the theoretical maximum distance which could be jumped.

Start vertical speed = 11sin45^o Start horizontal speed = 11cos45^o
Need to find time for jumper to return to ground.
Using $s = ut + ½at^2$ vertically:
$0 = 11\sin45^o t - 4.9t^2$
$0 = t(11\sin45^o - 4.9t)$
$t = 11\sin45^o/4.9 = 1.6$ *seconds (2 SF)*

Distance jumped: use horizontal motion
$s = ut$
$s = 11\cos45^o (1.58738) = 12$ *m (2 SF)*

Exam Hint *You may see very complicated equations in the textbooks but there is no point trying to remember them, just work through step by step using your basic equations.*

Typical Exam Question

A boy throws a ball with a speed of 40ms^{-1} at an angle of 30^o to the horizontal. Assuming the ball is projected from ground level, find
(a) the time taken for the ball to reach its greatest height
(b) the horizontal distance travelled by the ball

Start vertical speed = 40sin30^o Start horizontal speed = 40cos30^o

(a) Consider vertical motion:
Use $v = u + at$
$0 = 40\sin30^o - 9.8t$
$20 = 9.8t$
$t = 2.0$ *seconds (2 SF)*

(b) Here we must find the time taken for the ball to return to the ground first, then use this value to find the horizontal distance travelled.

Because the motion is symmetrical (the ball returns to the same height from which it started), this is ***double the time to the highest point****.*

So $t = 2(20/9.8)$
$s = 40\cos30^o (20/9.8) = 71m$ *(2 SF)*

Typical Exam Question

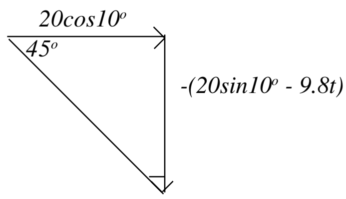
A girl stands at the top of a cliff, and throws a stone with a speed of 20ms^{-1} at an angle of 10^o above the vertical. It hits the sea 3 seconds later. Find
(a) the height of the cliff
(b) the highest distance above the cliff reached by the stone
(c) the time at which the velocity is at an angle of 45^o below the horizontal

Start vertical speed = 20sin10^o Start horizontal speed = 20cos10^o

(a) Using $s = ut + ½at^2$ vertically:
$s = 20\sin10t - 4.9t^2$
$s = 20\sin10 (3) - 4.9(3^2) = -34$
Height is 34m (2 SF)
NB: the negative sign just means the projectile has finished below its starting point.

(b) Using $2as = v^2 - u^2$ vertically
$2(-9.8)(s) = 0 - (20\sin10^o)^2$
$s = (20\sin10^o)^2/ 19.6$
$= 0.62m$
NB: we ***cannot*** *assume it reaches the highest point halfway through the motion, is the motion is not symmetrical - the stone ends up lower than it starts.*

(c) $v_v = 20\sin10^o - 9.8t$ $v_h = 20\cos10^o$



NB: we must include the negative sign on the vertical velocity, because we are considering its component downwards rather than upwards

So $\tan45^o = \dfrac{-(20\sin10^o - 9.8t)}{20\cos10^o}$

$\tan45^o = 1$ *so* $-(20\sin10^o - 9.8t) = 20\cos10^o$
$9.8t = 20\cos10^o + 20\sin10^o$
$t = 2.4$ *seconds (2SF)*

Qualitative test
1. State the vertical velocity of a projectile at its highest point.

2. Sketch the displacement-time and velocity-time graphs for
(a) an object thrown vertically upwards
(b) an object thrown vertically downwards

3. Sketch horizontal and vertical displacement-time graphs for an object projected horizontally from a window

Quantitative (calculation) test
1. Calculate the vertical velocity of an object thrown downwards at 23 ms^{-1} after 2.0 *s*.

2. A ball is thrown vertically upwards with speed *u*. It reaches a height of 15 metres. Calculate *u*.

3. A stone is dropped down a well. It hits the water 2.5 seconds later. Find the depth of the well.

4. An object is in the air for 3.6 *s*. It was launched from a tower horizontally with velocity 8 ms^{-1}
(a) how far away from the tower does it land?
(b) what angle with the horizontal does the final velocity have?

5. A stone is catapulted from ground level. Its initial velocity has horizontal component 30 ms^{-1} and vertical component 40ms^{-1}
Find:
(a) Its initial speed
(b) Its speed 3 seconds after it is projected
(c) The maximum height it reaches
(d) Its horizontal range

6. A stone is thrown with a speed of 20ms^{-1} at an angle of 40° to the horizontal. Five metres from the point from which it is projected there is a wall of height 4 metres.
Determine whether the stone will hit the wall.

7. A girl is attempting to throw a ball across her bedroom onto a shelf. The shelf is 2m above floor level and the horizontal distance from the girl to the shelf is 5m

The girl throws the ball with speed *u* ms^{-1} at an angle of 30° to the horizontal. It lands on the nearest edge of the shelf.

(a) Show the time of flight of the ball is $\frac{5}{u\cos 30^\circ}$

(b) Find the value of *u*

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Answers
1. $v = 23 - 9.8(2)$
$= 3.4\ ms^{-1}$

2. $2as = v^2 - u^2$
$2(-9.8)(15) = 0 - u^2$
$294 = u^2$
$u = 17\ ms^{-1}$ (2 SF)

3. $s = ut + ½at^2$
$s = -4.9t^2$
$= -4.9(2.5^2)$
$= -31$
Depth = 31m (2SF)

4. (a) Horizontal motion:
$s = 8(3.6)$
$s = 28.8$
$s = 29$m (2 SF)

(b) $v_v = -9.8(3.6) = -35.28$ $\quad v_h = 8$
Angle $= \tan^{-1}(35.28/8)$
$= 77°$ below horizontal

5. (a) $\sqrt{30^2 + 40^2} = 50ms^{-1}$

(b) $v_h = 30\ ms^{-1}$ $\quad v_v = 40 - 9.8(3) = 10.6\ ms^{-1}$
$v = \sqrt{30^2 + 10.6^2}$
$v = 32\ ms^{-1}$ (2SF)

(c) Using $2as = v^2 - u^2$ vertically
$2(-9.8)s = 0 - 40^2$
$s = 1600/19.6 = 82$ m (2SF)

(d) Using $s = ut + ½at^2$ vertically
$0 = 40t - 4.9t^2$
$0 = t(40 - 4.9t)$
$t = 40/4.9 = 8.1633$ seconds
So range = 30 (8.1633) = 240 m (2 SF)

6. $u_h = 20\cos 40°$ $\quad u_v = 20\sin 40°$

Need to find the height of stone after it has travelled 5m horizontally
Horizontal motion:
$5 = 20\cos 40° t$
$t = 5/(20\cos 40°)$

At this time, vertical height is given by:
$s = 20\sin 40° t - 4.9t^2$
$s = 20\sin 40° (5/20\cos 40°) - 4.9(5/20\cos 40°)^2$
$s = 3.67$ m
So stone will hit wall, since its height is less than 4 metres here

7. $u_h = u\cos 30°$ $\quad u_v = u\sin 30°$

(a) Using $s = ut$ horizontally:
$5 = u\cos 30° t$
$t = 5/u\cos 30°$

(b) When t = $5/u\cos 30°$, $s_v = 2$
$2 = u\sin 30°(5/u\cos 30°) - 4.9(5/u\cos 30°)^2$
$2 = 5\sin 30°/\cos 30° - 24.5/u^2\cos^2 30°$
$2 = 2.8868 - 32.6667/u^2$
$32.6667 = 0.8868u^2$
$u = 6.1\ ms^{-1}$

Physics Factsheet

www.curriculum-press.co.uk Number 72

Why Students Lose Marks: AS Motion Questions

This Factsheet analyses students' real answers to exam questions on motion in a straight line. By the end of this Factsheet, you should be more confident about:
- What the examiners want
- The kinds of things you are likely to be asked
- Common mistakes and misunderstandings

As you read the students' answers to the questions and the comments, try to work out what the student should have done - using the hints and comments if necessary - before you read the markscheme.

What do you have to know?

In this type of question, the examiner is trying to assess whether you can:
- understand the distinction between vector and scalar quantities (displacement vs distance and velocity vs speed)
- use the equations of motion
- draw, interpret and do calculations using graphs representing motion (displacement-time and velocity-time graphs)

In any question involving **calculation**, there are likely to be marks available for showing a clear **method.** If you have to use one answer in the next part of the question, there are likely to be "error carried forward" marks available - so you are only penalised once for a wrong answer.

Sally kicks a ball along the ground at a wall 3.0 m away. The ball strikes the wall at right angles, with a velocity of $6.0 ms^{-1}$ and rebounds in the opposite direction with an initial velocity of $4.5 ms^{-1}$. Sally stays in the same place, and stops the ball when it returns to her.

(a) Explain why the final displacement of the ball is not 6.0 m.

it's ended up in the same place it started

✓ *Mark awarded - just! - as the student has shown s/he knows what displacement means. But it would have been better to also explain that displacement is a vector so moving 3.0m forwards and 3.0m backwards results in a displacement of 0.*

[1]

(b) Explain why the average velocity of the ball is different from its average speed.

it isn't always moving in the same direction ✓✗

One of the two marks awarded - student should have realised this was insufficient for 2 marks. The examiner is looking to see that you know what average velocity and average speed are - so explaining how each is calculated would have been useful

[2]

(c) The ball is in contact with the ball for 0.15 seconds. Calculate its average acceleration during this period

$$\text{acceleration} = \frac{\text{change in speed}}{\text{time}}$$ ✗

*acceleration is the change in **velocity**, not speed. That means the direction is important - the change from $6.0 ms^{-1}$ forward to $4.5 ms^{-1}$ backwards is a change of $10.5 ms^{-1}$, not $1.5 ms^{-1}$*

$$= \frac{1.5}{0.15}$$

acceleration = $10 ms^{-2}$ ✓*ecf. The incorrect value for change in velocity has been used correctly to calculate an acceleration*

[2]

Hints and Comments

- *On "wordy" questions, use the number of marks to help you judge how much to write - for two marks, you must make two points*
- *In part (c), if the student had not shown working, it would not have been possible to award the "error carried forward" (ecf) mark - even if the examiner had guessed what the student had done.*
- *Questions involving distinctions between speed/velocity and distance/displacement are likely to involve calculations where you must take the direction into account. One way to do this is to put forward velocities as positive and backward ones as negative*

Markscheme

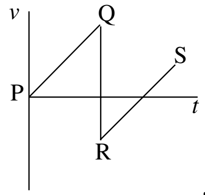
(a) displacement is a vector/ ball travels in opposite directions (1)

(b) velocity is rate of change of displacement (1) but speed is rate of change of distance (1)
or velocity is a vector/ speed is a scalar (1) and the velocity changes direction (1)

(c) change in velocity = $-4.5 - 6 = -10.5 ms^{-1}$ (1)
acceleration = $-10.5/0.15 = -70 ms^{-2}$ (1) *ecf*

Now try marking this student answer before you look at the markscheme and comments.

The graph below shows the variation of velocity with time for a vertically bouncing ball. The ball is released above the ground at P.



(a) What happens at point Q? *Velocity changes from positive to negative* [1]

(b) The gradients of lines PQ and RS are the same. Explain why.

The speed is the same from P to Q and from R to S [2]

(c) What is represented by the area between line PQ and the time axis?

distance [2]

(d) The ball is dropped from a height of 0.80m. Calculate its speed immediately before impact (*assume* $g = 9.8\ ms^{-2}$)

$2as = v^2 - u^2$ $20 \times 0.8 = v^{-1}$ $v = 4\ ms^{-1}$ [2]

Here's the markscheme:

(a) The ball hits the floor (1)

(b) The gradient represents acceleration (1) which is constant because it is the acceleration due to gravity (1)

(c) height/distance/displacement (1) of the ball above the ground when it is dropped (1)

(d) $v^2 = 2 \times 9.8 \times 0.8 = 15.68$ (1)

$v = 4.0\ ms^{-1}$

So how did the student score?

(a) *Velocity changes from positive to negative* 0/1 - although this is true, "what happens" refers to something physical that you would observe - so you have to refer to the ball changing direction - or better (read the question!) bouncing

(b) *The speed is the same from P to Q and from R to S* 0/2 -the student appears to have confused this with a displacement graph (where the gradient would correspond to the velocity)

(c) ***distance*** 1/2 - correct, but not specific enough about **which** distance. The student has not used the fact that the line PQ is mentioned - the answer given is a general answer to what is represented by the area under a velocity-time graph

(d) *$2as = v^2 - u^2$ $20 \times 0.8 = v^2$ $v = 4\ ms^{-1}$* 1/2 - correct method, but the candidate has not used the value of g given in the question.

A car accelerates uniformly from a speed of $5.0\ ms^{-1}$ to a speed of $13\ ms^{-1}$ in 4.0 seconds.

(a) Calculate its acceleration

(13 - 5)/4 = 2 ✓ ✗ A correct calculation, but the student has omitted the units, and despite all the original data being given to 2 SF, has only quoted 1 SF in the answer [2]

(b) Calculate the distance it travels in this time

9×4 = 36m ✓ ✓ Both marks awarded - but this was a risky strategy, is it is not clear what the "9 ×4" refers to without a formula being quoted (average speed ×time = distance, in this case) [2]

(c) Explain why the word "uniformly" in the question is important

couldn't use the equations otherwise ✗ Although this is true, the student has not explained fully enough - or demonstrated an understanding of the word "uniformly" [1]

Hints and Comments

- *Always make it clear which formula you are using to ensure you get method marks*
- *It does matter whether you write 2 ms^{-2} or 2.0 ms^{-2} - the zero shows it is accurate to 2 SF. That's why questions write it like this.*

Markscheme

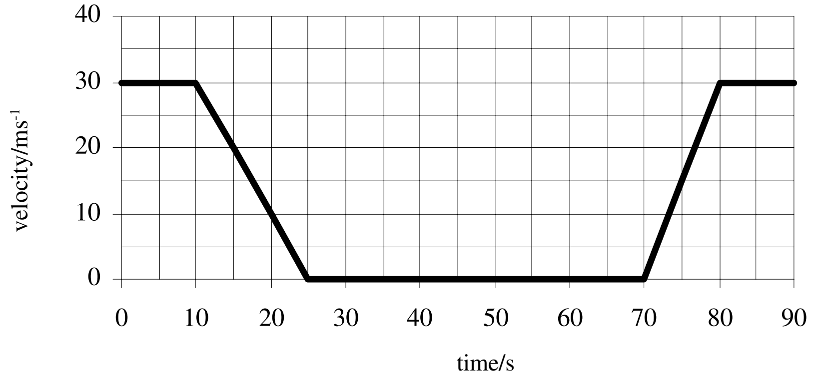
(a) a = (13 - 5)/4 (1) a = 2.0 ms^{-2} (1)

(b) Using $2as = v^2 - u^2$ or $s = ut + \frac{1}{2}at^2$ or $s = \frac{1}{2}(u + v)t$ (1)

36m (1)

(c) This means constant acceleration - which is required for the use of the standard equations (1)

The diagram shows a velocity-time graph for a car that stops at traffic lights then moves away.



(a) Use the graph to show that the car travels 225m while it is decelerating

½(30)(15) = 225m ✗✓ *The question indicated that the graph had to be used - the candidate has shown no evidence of this. One mark awarded for correct calculation* [2]

(b) Calculate the acceleration of the car after it moves away

30/10 = 3 ✓✗ *Correct calculation and answer, but no units* [2]

(c) The velocity of a second car is measured over the same period of time.
Its initial velocity is 40 ms^{-1}, which it maintains for 10 seconds. It then decelerates uniformly to rest at 4ms^{-2}.
It remains at rest for 40 seconds, then accelerates uniformly to a velocity of 30ms^{-1} while it covers the next 300m.
It then maintains a constant velocity of 30 ms^{-1} during the rest of the time for which it is observed.

(i) Calculate the time for which the car is decelerating

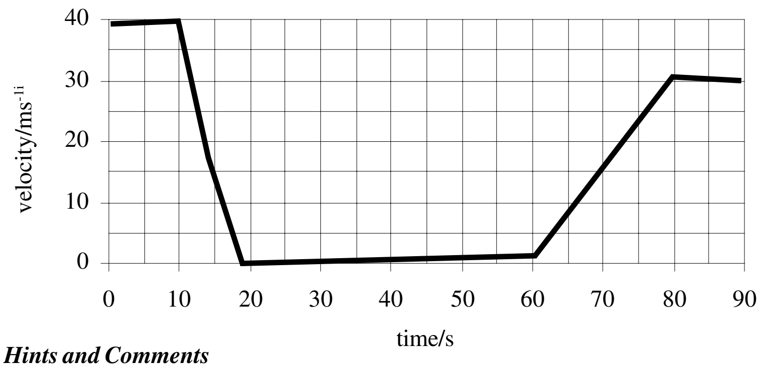
10 s ✓ *Correct - but why no working? When a "calculation" is asked for, working is expected* [1]

(ii) Calculate the time for which the car is accelerating after being at rest

300 = ½at²   600 = at²
30 = at   30 × t = 600   t = 20

✓✓ *Full marks here - but this is a very hard method! Perhaps the candidate had not remembered the equation s = ½ (u + v)t, which would have given the answer much more easily* [2]

(iii) Draw the velocity time graph for this car on the axes below. [2]



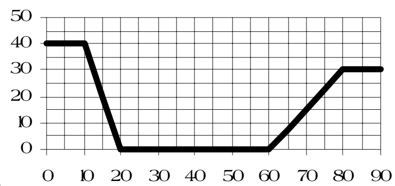
✗✓ *The candidate clearly understood what was required, but has lost a mark through sloppy drawing - the parts that should be horizontal do not appear to be so, the times are not exact and one of the lines looks more like a curve.*

Hints and Comments

- *If you are asked to "show that", you need to be extra careful to show working. If you are told to use a graph, make sure you either show working on the graph (eg marking a triangle) or state how you are using it (eg area under graph = distance)*
- *You may find it helpful to write out all the equations of motion before starting a question - then it is easier to choose the appropriate one*
- *Graphs do have to be drawn accurately - use a ruler where appropriate, and make curves smooth. "Fudging" it if you are not sure of the exact point will definitely not get you the mark.*

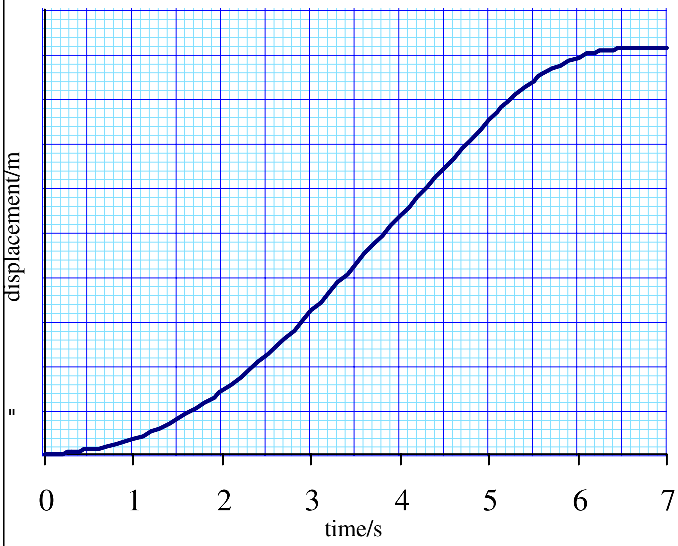
Markscheme

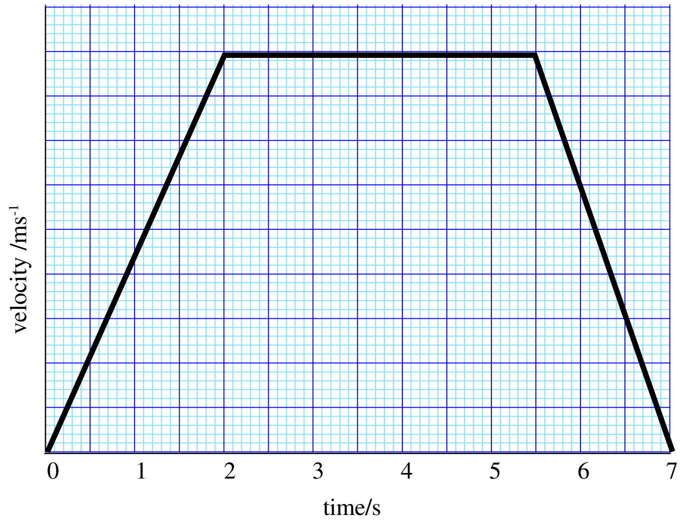
(a) Area under graph = displacement or evidence of using graph (1)
½ ×30× 15 = 225m (1)
(b) 30/10 (1)   = 3ms^{-2} (1)
(c) (i) 40/t = 4   t = 10 s (1)
(ii) 300 =½ (0 + 30)t (1)   t = 20s (1)
(iii) Correct shape (1) Points plotted correctly (allow e.c.f. from above)



Now try marking this student answer before looking at the markscheme.

Jaz is running along a straight track. The graph below on the left shows how his displacement varies with time. Without calculation, sketch on the graph paper on the right how his velocity varies with time. [4]





Hints and Comments

- *If the displacement-time graph looks like part of a **quadratic** curve, that means the velocity is increasing linearly.*
  If the displacement-time graph is a straight line, the velocity is constant
  If the displacement-time graph is horizontal, the velocity is zero.
- *Always look out for key points in time when the motion changes, and line them up*

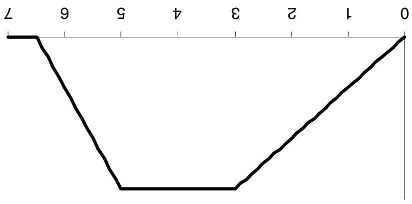
Markscheme

(linear) increase to t ≈ 3 (1)

constant velocity from t≈ 3 to t ≈ 5 (1)

(linear) decrease from t≈ 5 to t ≈ 6.5 (1)

zero velocity from t ≈ 6.5 onwards (1)



So how did the student score?

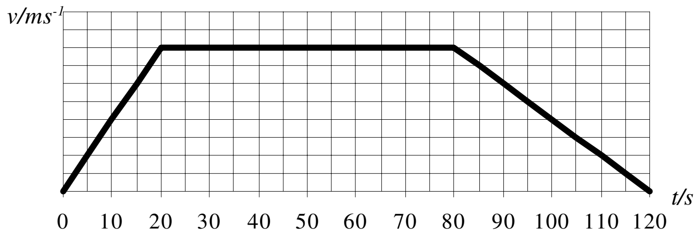
Although the graph is broadly the right shape and looks similar to the markscheme, the student omitted the zero velocity part altogether - and the times were too inexact to score any of the other marks. So 0/4.

Questions

1. A car accelerates uniformly from rest for 10 seconds. In that time it covers 200m. Calculate its acceleration. [2]

2. Explain how it is possible to travel with constant speed, but varying velocity. [2]

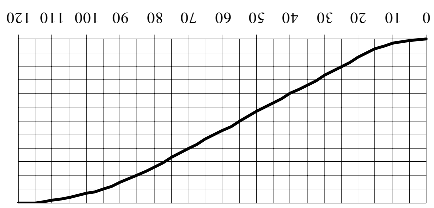
3. The diagram below shows a velocity-time graph for a runner. Without doing calculations, sketch the displacement-time graph [1]



4. Explain how the graph in question 3 could be used to calculate
   (i) the distance covered by the runner [1]
   (ii) the acceleration and deceleration of the runner [1]

Answers

1. $s = \frac{1}{2}at^2$ $200 = \frac{1}{2}a(100)$ (1)
   $a = 4ms^{-2}$ (1)
2. Velocity is a vector/velocity includes a direction (1)
   So you could change direction but not speed (eg move in a circle) (1)
3. quadratic increase from 0 to 20 (1)
   straight line increase 20 to 80 (1)
   rate of increase slows (quadratic again) from 80 to 120 (1)



4. (i) Area between graph and time-axis
   (ii) Gradient of the graph

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Physics Factsheet

September 2000 Number 02

Vectors and Forces

Vector quantities are commonly encountered in physics. This Factsheet will explain:
- the difference between a vector and a scalar, giving common examples of each
- how to add and subtract vectors
- how to resolve vectors
- how to apply this to forces and particles in equilibrium

1. What is a vector?

- *A vector is a quantity that has both **magnitude** and **direction**.*
- *A **scalar** just has a magnitude – it is just a (positive or negative) number.*

A vector can be represented by an arrow – the length of the arrow represents the magnitude of the vector, and the direction in which the arrow is pointing shows you the vector's direction.

Anything that has a direction as well as a size will be a vector – for example, if you tell someone that your house is 200m East of the chip shop, you are representing the position of your house by a vector – its magnitude is 200m and its direction is East. If, instead, you just said that your house was 200m away from the chip shop, you are using a scalar quantity (distance). You will notice that the scalar is not so useful as the vector in telling someone where your house is!

Table 1 shows some common examples of scalar and vector quantities.

Table 1. Scalars and Vectors

Scalars	Vectors
Distance	Displacement
Speed	Velocity
Temperature	Acceleration
Energy	Force
Power	Momentum
Pressure	Torque/Moment
Mass	Impulse

Exam Hint: *Exam questions often require you to explain the difference between a scalar and a vector, and to indicate whether a particular quantity is a vector or scalar. If you think you might find it hard to work this out in an exam, make sure you learn the common examples.*

It is particularly important to understand the distance between distance and displacement, and between speed and velocity (and hence average speed and average velocity):

Suppose you walk 5m North, then 3m South. The total **distance** you have travelled is obviously 8m. However, your final **displacement** – which just means where you end up relative to where you started – is 2m North.

Suppose you were walking at a steady 2 ms^{-1}. Then your **speed** was constant throughout – it was 2 ms^{-1}. However, your **velocity** was not constant, since to begin with you were travelling at 2ms^{-1} North, then you changed to 2ms^{-1} South.

Your **average speed** for the walk is:

total distance÷total time = 8÷4 = 2ms^{-1}

(since it takes 5÷2 = 2.5s walking North and 3 ÷2 = 1.5s walking South).

However, your **average velocity** is:

total **displacement** ÷total time = 2m North ÷4 = 0.5ms^{-1} North

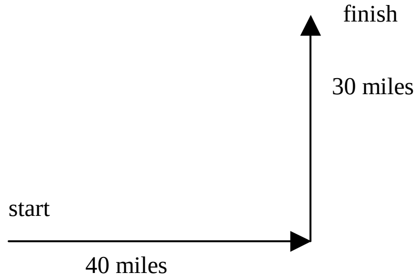
Tip: *Always be very careful not to use "speed" when you mean "velocity" and vice versa – ask yourself first whether it has a direction.*

Typical Exam Question
a) State the difference between scalar and vector quantities [1]
b) Give two examples of a vector quantity. [2]

a) a vector has direction ✓ a scalar does not
b) any two examples from vector column in table 1 ✓✓

2. Adding Vectors

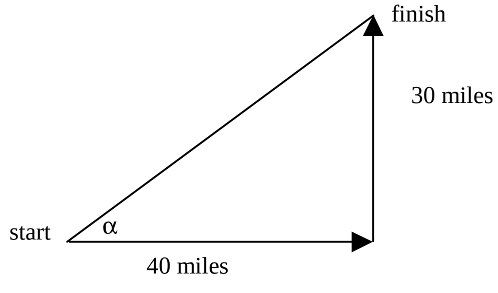
Imagine driving 40 miles East then 30 miles North. Your path would look like:



How far away from your starting point have you ended up? It certainly isn't 70 miles. If you draw the other side of the right-angled triangle, and use Pythagoras' Theorem, you will find it is 50 miles.

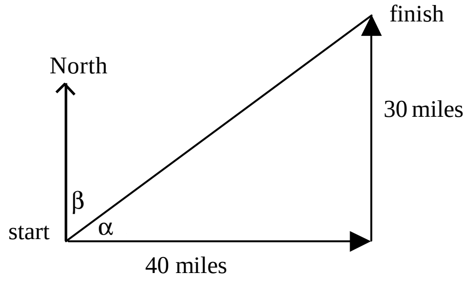
To describe the position at which you ended up, you'd say it was 50 miles from your starting point, but you'd also need to say *in what direction*. It won't be a direction like Northeast (since that would mean going the same distance North and East), so we'd have to give the direction in terms of an angle.

You might choose to find the angle marked α in the diagram below.



Using trigonometry (see the Factsheet Maths for Physics: Trigonometry if you need help on this), we find $\alpha = \tan^{-1}(0.75) = 36.9^{o}$.

But it is more conventional to give directions as a **bearing** – measured clockwise from North. So we'd need the angle marked β in the diagram below:



We can see that $\alpha + \beta = 90^o$, so $\beta = 53.1^o$.

We can now describe the final displacement as:
50 miles at a bearing of 53.1^o

The example above was an example of **vector addition**. We added two displacements (40 miles East and 30 miles North) to find a **resultant** diplacement (50 miles at a bearing of 53.1^o). In this case, we found the resultant by calculation (using Pythagoras' Theorem and trigonometry). It could also have been found by **scale drawing**.

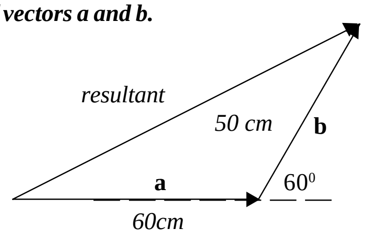
> ***Tip:*** *In any problem of this type, you MUST draw a diagram – even if it is only a sketch.*

Finding the resultant of two vectors
To find the resultant of two vectors:
- Draw one of the vectors
- Draw the other vector starting at the end of the first one.
- Draw an arrow from the start of the first vector to the end of the second one – this represents the resultant
- Use calculation or accurate measurement (if you are told to use scale drawings) to find the length (magnitude) of the resultant.
- Find the direction of the resultant

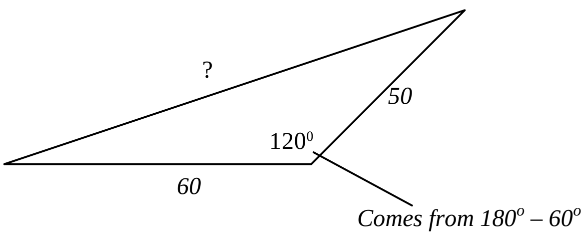
If you are using calculation, you may need to use the sine and cosine rules as well as trigonometry in normal right-angled triangles. If you do not like this sort of trigonometry, you may prefer to use the alternative method given at the end of section 3 in this Factsheet.

> ***Exam Hint:*** *If you are asked to find the resultant, you* ***must*** *give its direction as well as its length if you want full marks*

Example 1. Vector a is of magnitude 60cm and acts horizontally. Vector b is of magnitude 50cm and acts at 60º above the horizontal. Find the resultant of vectors a and b.



First find the magnitude of the resultant; to do this we use this triangle:

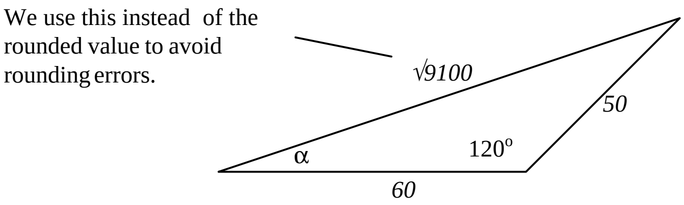


Use the cosine rule:
$a^2 = b^2 + c^2 - 2bc\cos A$; $\quad A = 120^o, a = ?, b = 60, c = 50$

$a^2 = 60^2 + 50^2 - 2 \times 60 \times 50 \times \cos 120^o$
$= 3600 + 2500 - 6000\cos 120^o$
$= 6100 + 3000 = 9100$
$a = \sqrt{9100} = 95$cm (nearest cm)

> ***Tip:*** *When you are using the cosine rule, take care with "BIDMAS" – you must work out 2bc cosA, and subtract the answer from $b^2 + c^2$. Also, always check that your answer sounds sensible – if it does not, you may have forgotten to square root.*

Now we need to find the direction. Since in the question angles are given to the horizontal, we should give the answer in that way. So we want angle α shown below:



Use the sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
$A = 120^o, a = \sqrt{9100}, b = 60, c = 50, C = \alpha$

> ***Tip****: You can write the sine rule with all the sines on the top, or with all the sines on the bottom. If you are trying to find an angle, have them on the top, and if you are trying to find a side, have them on the bottom.*

We leave out the part involving sinB, since we are not interested in it, and it is not useful. So we have:

$\frac{\sin 120}{\sqrt{9100}} = \frac{\sin\alpha}{50}$

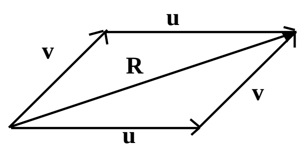
$\frac{\sin 120}{\sqrt{9100}} \times 50 = \sin\alpha$

$0.45392... = \sin\alpha$

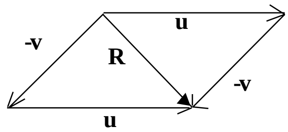
$27^o = \alpha$ (nearest degree)

So the resultant has magnitude 95cm and is at an angle of 27º above the horizontal.

Parallelogram rule of vector addition
Let us consider adding two vectors, **u** and **v**, to give their resultant, **R**. **R** is the **diagonal** of the **parallelogram** whose sides are **u** and **v**. This gives another visual way to think about vector addition.



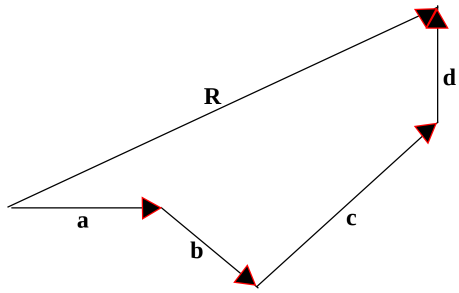
The parallelogram can also be used to **subtract** vectors - to find **u - v**, you'd draw the parallelogram using vectors **u** and **-v**:



Note that for calculation purposes, the parallelogram works exactly the same way as the vector triangle.

Resultant of more than 2 vectors

To find the resultant of more than 2 vectors, we draw a **vector polygon**. This is very similar to finding the resultant of 2 vectors, except that you draw the third vector after the second vector, and so on. The diagram shows the vector addition of vectors **a**, **b**, **c** and **d**, and their resultant, **R**.



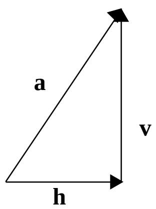
If you need to find the resultant of 3 or more vectors by calculation, it is best to use the method described at the end of section 3.

> ***Special case**: If the vector polygon **closes** (so the end of the last vector coincides with the start of the first one), then the resultant is **zero**.*

> ***Tip**: It does not matter in which order you add vectors, so if it makes your diagram or calculation easier to put them in a particular order, go ahead!*

3. Resolving

Resolving a vector involves writing it as the sum of other vectors – it's like resolving in reverse. For example, the vector **a** shown below can be written as the sum of a horizontal vector (**h**) and a vertical vector (**v**).



The seperate vectors that the original is resolved into are called **components** – in the above example, **h** is the horizontal component of **a** and **v** is the vertical component of **a**.

Note that there are many other ways we could resolve vector **a** – we choose the most convenient way in each situation (see section 4 for some examples of this). It is, however, always best to resolve a vector into two **perpendicular** components.

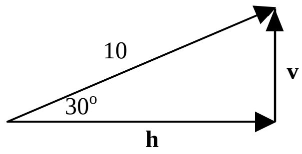
> ***Exam Hint:** In practical problems, the two directions are usually horizontally and vertically, or if an inclined plane is involved, along and perpendicular to the plane.*

Calculating components

To find the components of a vector in a pair of perpendicular directions, we will be using a right-angled triangle.

- Take the vector as the hypotenuse of the triangle
- Take the directions you want to resolve in as the other two sides
- Use trigonometry to work out the size of the components.

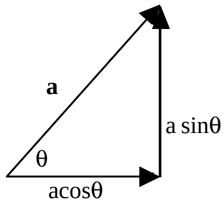
Example 2. A vector of magnitude 10 is inclined at 30^o above the horizontal. Find the horizontal and vertical components of the vector.



Using trigonometry: $\cos 30^o = \frac{h}{10}$, so $h = 10 \times \cos 30^o = 8.66$

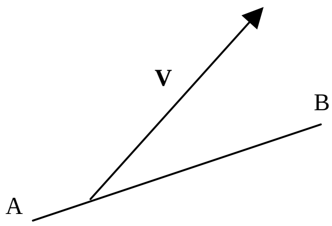
$\sin 30^o = \frac{v}{10}$, so $v = 10 \times \sin 30^o = 5$

In fact, you can save time working out the trigonometry by remembering the following diagram showing the vector **a** and its components:



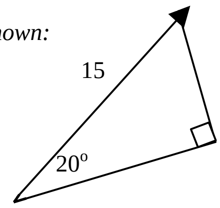
The following example shows how to apply this.

Example 3. The diagram below shows a vector V and the line AB.

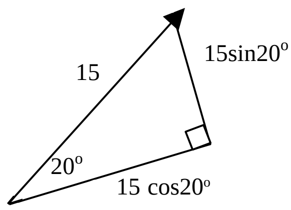


The angle between V and AB is 20^o. Find the components of V parallel and perpendicular to AB, given that the magnitude of V is 15.

Our triangle is as shown:



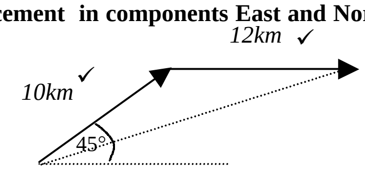
By comparison with the diagram above, we get:



So the component parallel to AB is 15cos20^o = 14 (2 SF)
The component perpendicular to AB is 15sin20^o = 5.1 (2SF)

Typical Exam Question
Bill travels 10km North-east and then 12km due East
(a) Draw a vector diagram showing Bill's route. [2]
(b) Calculate, without the use of a scale diagram, Bill's resultant displacement in components East and North. [3]

(a)



(b) E: 10cos45 ✓+ 12✓ = 19km. N: 10sin45 = 7.1km✓

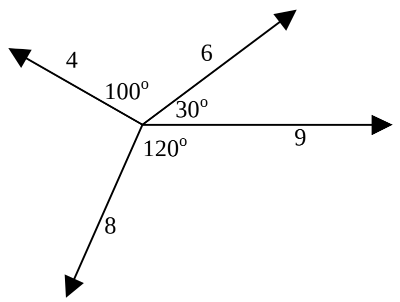
Using components to find the resultant

For problems involving many vectors, the following provides a fail-safe method to find the resultant force:

- Choose two sensible perpendicular directions – horizontal and vertical are often a good idea.
- Resolve every vector involved in these two directions (you could put them in a table to aid clarity and ensure you have not missed one out)
- Add up all the components in one direction (say horizontal), which gives the resultant force in that direction.
- Repeat for all the components in the other direction.
- You are now left with two perpendicular vectors. Find the resultant of these two vectors using Pythagoras and basic trigonometry.

> ***Exam Hint****: A common mistake is to resolve correctly, but ignore the direction of the component – for example, 6 units upwards is not the same as 6 units downwards! To avoid confusion, decide at the beginning which direction to take as positive.*

Example 4. Find the resultant of the vectors shown below.



We will take to the left, and upwards as positive

Vector	*Horizontal compt*	*Vertical compt*
9	9	0
6	6 cos30°	6sin30°
4	-4cos50°	4sin50°
8	-8cos60°	-8sin60°

The angles 50° and 60° come from using angles on a straight line = 180°

> ***Tip:*** *Do not actually work out the sines and cosines yet, to avoid rounding errors or copying errors.*

So total of horizontal components $= 9 + 6\cos30^o - 4\cos50^o - 8\cos60^o$
$= 7.625...$

total of vertical components $= 0 + 6\sin30^o + 4\sin50^o - 8\sin60^o$
$= -1.864...$

So to find overall resultant:

By Pythagoras, magnitude of $R = \sqrt{(7.625^2 + 1.864^2)} = 7.8$ *(2 SF)*

$\alpha = \tan^{-1}(1.864 \div 7.625) = 14^o$ *below the horizontal (2 SF)*

4. Application to forces and equilibrium

To find the resultant of a number of forces, use the methods described above.

A body is in **equilibrium** if there is no resultant force (and no resultant torque – see Factsheet 4 Moments and Equilibrium) on it. In the examples considered in this Factsheet, there will never be a resultant torque, so we will only have to use that the resultant force is zero.

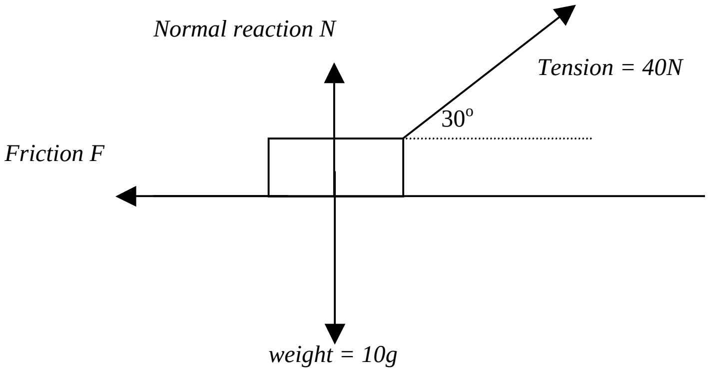
Any body is in equilibrium if it is at rest, or moving with constant velocity (**not** just a constant speed) – see Factsheet 12 Applying Newton's Laws for more details on this.

Problems involving equilibrium often require you to use the fact that the body is in equilibrium to find unknown forces. The procedure here is:

- First read the question carefully to check whether the particle is in equilibrium
- Draw a diagram showing all the forces
- Resolve all forces in two perpendicular directions
- Find the total of the components in each direction, and equate it to 0.
- Solve your equations to find any unknown forces.

Example 5. A box of mass 10kg is being towed at constant velocity along rough horizontal ground by a rope inclined at 30° to the horizontal. The tension in the rope is 40N. Find:
a) the frictional force exerted by the ground on the box
b) the normal reaction force exerted by the ground on the box.
Take g = 9.8ms⁻²

We know the box is in equilibrium because it is moving with ***constant velocity***

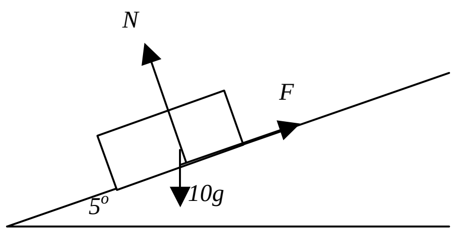


Resolve horizontally and vertically (taking upwards and left as positive), and equate to 0:

$\rightarrow \quad 40\cos30^o - F = 0$
$\uparrow \quad 40\sin30^o + N - 10g = 0$

From the first equation, we get $F = 40\cos30^o = 35N$ *(2 SF)*
From the second equation, we get $N = 10g - 40\sin30^o = 78N$

Example 6. A box of mass 10kg is at rest on a rough plane inclined at 5° to the horizontal. Find the normal reaction and the frictional force exerted by the plane on the box. Take g = 9.8 ms⁻²



The box is in equilibrium as it is at rest.
Note that friction must act up the plane, since the box will be "trying" to slide down.

When there is an inclined plane involved, it is best to resolve parallel and perpendicular to the plane.

The weight of the box acts at an angle of 85° to the plane (from angles in a triangle). So we have:

Along the plane: $F - 10g\cos85^o = 0 \Rightarrow F = 8.5N$ *(2SF)*
Perpendicular to the plane: $N - 10g\sin85^o = 0 \Rightarrow N = 98N$ *(2SF)*

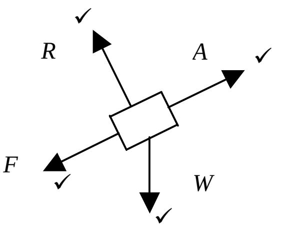
Typical Exam Question

A body of mass 5.0kg is pulled up a rough plane, which is inclined at 30° to the horizontal, by the application of a constant force of 50N, which acts parallel to the plane. Take $g = 9.8\ ms^{-2}$

(a) Draw a free body diagram for the body showing the normal reaction R, frictional force F, weight W and the applied force A. [4]

(b) When the arrangement is in equilibrium, what are the values of the normal reaction force R and the frictional force F? [4]

(a)



(b) Resolve forces perpendicular to the plane:

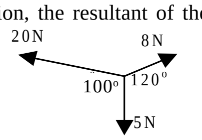
R = Wcos30° ✓ *so R = 42.4 ≈ 42N* ✓

Resolve forces parallel to the plane:

F + Wsin30° = 50 ✓ *so F = 21.7 ≈ 22N* ✓

Questions

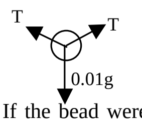
1. a) Explain the difference between a vector and a scalar
   b) Indicate whether each of the following is a scalar or a vector:
   Density Momentum Electrical resistance Distance Acceleration
2. Find the resultant of each of the following:
   a) A force of 5N acting horizontally to the left and a force of 8N acting vertically upwards
   b) A force of 10N acting vertically upwards and a force of 3N acting at 20° to the upward vertical.
   c) A force of 5N acting at 10° below the horizontal and a force of 5N acting at 10° above the horizontal
   d) A force of 6N acting horizontally to the left, a force of 8N acting vertically upwards and a force of 2N acting horizontally to the right.
3. A body is in equilibrium. Which of the following **must** be true? There may be more than one correct answer.
   a) the body is stationary
   b) the polygon of forces for the body is closed
   c) there are no forces acting on the body.
4. Find, by calculation, the resultant of the forces shown below :



5. A smooth bead of mass 10g is threaded onto a thin piece of string of length 2m. The ends of the string are fastened to the ceiling so that they are at the same horizontal level as each other and are 1.6m apart.
   a) Draw a diagram to show the forces acting on the bead.
   b) Explain why the bead only rests in equilibrium at the midpoint of the string.
   c) Find the tension in the string, taking g = $10ms^{-2}$

Answers

1. a) A vector has magnitude and direction; a scalar has magnitude only
   b) scalar, vector, scalar, scalar, vector
2. a) magnitude 9.4N, at 58° above horizontal
   b) magnitude 13N, at 4.6° to upward vertical
   c) 9.8N acting horizontally
   d) 8.9N acting at 63° above (leftward) horizontal
3. b) only
4. 13N at 11° above rightward horizontal
5. a)



   b) If the bead were not at the midpoint, the angles made by the two pieces of string would not be equal. If the angles were not equal, there would be a net horizontal force on the bead,since the horizontal components of the tensions would not balance, so the bead would not be in equilibrium.
   c) 0.083N (2SF)

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

A boat travels at $6.8ms^{-1}$ South-easterly towards a harbour, which is 10km away. Once the boat reaches harbour, the passengers get in a car and drive due North at $80kmhr^{-1}$ for 15 minutes. Calculate the:

(a) total distance travelled. [2]

d = s × t = 80 × ¼ = 20 + 10 = 30 ✓

1/2

> 1 mark deducted for omission of units.
> Although it would not lose marks, the candidate should avoid writing 80 × ¼ =...= 30, since it is not mathematically correct and could lead to confusion

(b) total displacement. [5]

$20^2 - 10^2 = 300$.

√300 = 17.3 km

0/5

> Candidate has attempted to treat this as a right-angled triangle, when it is not – either the cosine rule or resolving into components should have been used. Candidate has also not given the direction

(c) average speed. [2]

$80kmh^{-1} = 80 \times 1000/3600 = 22.2ms^{-1}$

So average speed = (22.2 + 6.8) ÷ 2 = $14.5ms^{-1}$

0/2

> Candidate has not used the correct definition of average speed – it is total distance/ total time, **not** the average of the individual speeds.

(d) magnitude of the average velocity. [2]

$14.5ms^{-1}$

0/2

> Poor exam technique – the candidate should appreciate that 2 marks will not be awarded for simply writing down the same answer again. Although the candidate's answer for total displacement was incorrect, credit would have been awarded if it had been used correctly to find average velocity.

Examiner's Answers

(a) Total distance = 10 + (80 × ¼) ✓ *= 30km* ✓

(b) Total displacement = 10km SE + 20km N

East = 10 cos45

North = -10 sin45 + 20.

= (7.1, 12.9) km ✓

Magnitude is $|R| = \sqrt{(7.1^2+12.9^2)}$ ✓ *=14.7 km*

Direction is given by $\theta = \tan^{-1}(12.9 / 7.1)$ ✓ *= 61.2°*

Displacement vector = 15km, ✓ *61° N of E (or bearing 029)* ✓

(c) Average speed = total distance / total time

= 30 km/ (25 + 15) minutes ✓

= 0.75 km / minute = 45 km / hr ✓

(d) Magnitude of average velocity

= (magnitude of total displacement) / total time

= (14.7 km)/ (⅔ hr) ✓ *= 22 km / hr* ✓

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Physics Factsheet

January 2001 Number 12

Applying Newton's Laws

What are Newton's Laws?

Newton's First Law: Every body continues at rest or with constant velocity unless acted upon by a resultant force

Newton's Second Law: The rate of change of momentum of a body is proportional to the resultant force that acts on it

Newton's Third Law: When two bodies interact, the forces they exert on each other are of equal magnitude and opposite direction

1. The First Law

This is mainly used to find unknown forces, by using it in the form:

If a body is at rest or moving with constant velocity, there must be no resultant force on it.

Tip: *It is important to note that the body must be moving with constant* ***velocity****, not just constant* ***speed*** *– the body must move at a steady speed and always in the same direction. A body moving in a circle may move at a steady speed, but still has a resultant force on it.*

A body that is at rest or moving with constant velocity is said to be in **equilibrium**. Factsheets 2 Vectors and Forces and 4 Moments and Equilibrium give details on how to solve problems about bodies in equilibrium, but the general strategy is:

- ♦ Draw a diagram, showing **all** the forces on the body.
- ♦ Check the body really is in equilibrium– is it stationary or moving with constant velocity?
- ♦ Resolve forces in two perpendicular directions – either horizontally and vertically, or if the body is resting on a slope, then parallel and perpendicular to the slope.
- ♦ Equate the total force in each direction to zero.
- ♦ If necessary, take **moments** – this will be required if not all the forces pass through one point – and equate to zero.
- ♦ Solve your equations to find the unknown forces.

We can also use the First Law to help with problems for bodies that are not in equilibrium, provided there is no resultant force on them **in a particular direction**. In cases like this it is used together with the Second Law – see later for examples.

Typical Exam Question
An aircraft of mass 11 000kg, which moves at a constant velocity, v, and constant altitude, is powered by propellers and experiences a drag force.
(a) Draw a labelled free body diagram showing the 4 main forces acting on the aircraft. [4]
(b) The thrust from the propellers is 225kN and the drag force is given by $10v^2$. Calculate the aircraft's level flight speed. [2]

(a) Lift vertically upward ✓ weight vertically downward ✓
thrust forwards ✓ drag backwards ✓
(b) For level flight, horizontal forces are equal:
$225000 = 10v^2$ ✓
$v = 150ms^{-1}$ ✓

Free-Body Force Diagrams

Many questions require you to draw a free-body force diagram; however, even if it is not asked for explicitly, it is **vital** in solving any question involving Newton's Laws. All the diagram shows is the body in which you are interested, together with all the forces acting on it – **not** the forces acting on any other body.

It is a good idea to draw a diagram of the whole situation first, including all the bodies involved, since it helps you to make sure you do not miss out any forces – this is the commonest mistake!

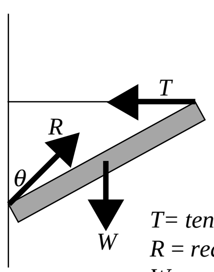
To avoid missing out forces, work through the following check-list:

- ♦ The body's weight, acting from its centre of mass.
- ♦ If the body is in contact with anything, there will be a normal reaction force. This acts at right-angles to the surface.
- ♦ The reaction at a hinge can act at any angle to the surface, so put it in at an unknown angle.
- ♦ Friction will act if the body is in contact with a rough surface, and if it is moving or has any "tendency" to move. Having a "tendency" to move means the body would move if there was no friction. Friction acts parallel to the surface, in the direction opposite to the way the body moves or would tend to move.
- ♦ If the body is attached to a string, the tension of the string will act on it. Tension always pulls, never pushes.
- ♦ If the body is attached to a spring, the tension or thrust of the spring will act on it. It can pull (tension) or push (thrust).
- ♦ If the body is moving through the air, then air resistance will act in the opposite direction to the one it is moving in.
- ♦ Aircraft experience a lift force vertically upwards.

Example 1. A uniform rod AB is attached at end A to a vertical wall by a hinge. A spring has one end attached to end B of the rod, and the other is fixed to the wall above A so that the spring is horizontal. Draw a free body force diagram to show the forces acting on the rod.

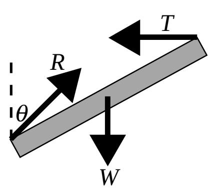
Note: the rod being "uniform" means that its centre of mass is at its centre.

Diagram of the whole thing



T= tension of spring
R = reaction at hinge
W = weight of rod

Free-body force diagram



Tip: *You must make it clear what any letters you use in your diagrams stand for.*

Typical Exam Question

A speedboat is towing a paraglider at a constant speed and height on the end of a light rope of length 30m, which makes an angle θ with the horizontal. The forces acting on the paraglider are the vertical lift, L, the horizontal drag, D, his weight, W and the tension in the rope, T.

(a) Draw a free body diagram of the paraglider showing the forces L, D, W and T. [2]

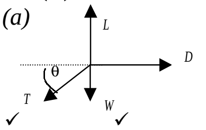
(b) State the value of the resultant of these forces. [1]

(c) Hence, write an equation relating the magnitudes of:

(i) D, T and θ [1]

(ii) L, W, T and θ. [1]

(a)



(b) The resultant of these forces must be zero. ✓

(c) (i) Resolving horizontally: $D = T\cos\theta$ ✓

(ii) Resolving vertically: $L = W + T\sin\theta$ ✓ .

Mass and Weight

It is vital to remember the difference between mass and weight. To summarise:

	Mass	Weight
Measures	The amount of matter in a body	The force of gravity on a body
Changes	Not at all, unless the body is broken up	Different depending on the force of gravity – so would be different on the moon, and high above the earth's surface
Unit	kilogram	newton
Scalar/ vector	scalar	vector (since it is a force)

For any body, W = Mg, where W is its weight (N), M is its mass (kg) and g is the acceleration due to gravity.

Weight always acts vertically downwards from the centre of mass of the body.

2. The Third Law

We are considering the third law next because a good understanding of both first and third law is necessary to approach some second law problems. Here, we will be applying the third law to bodies in contact, or connected by a rope. It tells us that, for example:

- the downward force you exert on the floor by standing on it is the same size as the upward force the floor exerts on you
- if you walk a dog on a lead, the tension in the lead acting on you (due to the dog tugging you) is of the same magnitude as the tension in the lead acting on the dog, tugging it towards you.
- when you push on a door, the door pushes back on you with the same magnitude force.

Example 2. Two children are playing "tug-of-war". One of them suddenly lets go of the rope. Explain why the other child may fall over, and explain the direction in which s/he falls.

When both children are tugging the rope, the force each child exerts on the rope is equal and opposite the force the rope exerts on the child – the rope pulls each child forward, and the child tries to pull the rope backwards. When one child lets go, the tension in the rope is removed. The other child is still exerting the same backward pull on the rope, but there is now no compensating forward pull on the child. So the child falls backwards.

Typical Exam Question

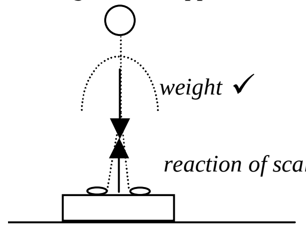
(a) Identify three properties of pairs of forces that are linked by Newton's third law. [3]

(b) A person stands on bathroom scales on the ground. Draw a free-body force diagram for the person. Identify all forces clearly. [2]

(c) For the situation in (b), state the other force forming a Newton's third law pair with the reaction force of the scales acting on the person's feet. [1]

(a) equal in magnitude ✓ *opposite direction* ✓ *act on different bodies* ✓

(b)



(c) The person's weight ✓

3. The Second Law

Newton's Second Law is most often used in the form

F = ma

F = resultant force (newtons) m = mass (kilograms)

a = acceleration (metres seconds^{-2}*)*

See Factsheet 9 Momentum for more on using the second law in its other form.

This form is **only valid if mass is constant**. However, since the examination does not require you to consider variable mass, this is not a problem.

Note that both F and a are **vectors** – the force determines not only the **size** of the acceleration, but also its **direction** – a body accelerates in the direction of the resultant force on it.

Second law problems – like any other mechanics problem – require you to draw a clear diagram, including all the forces acting on a body. You then need to:

- Resolve forces in the direction in which acceleration is taking place, and use F = ma, where F is the **resultant force** in that direction.
- If necessary, resolve forces **perpendicular** to the direction of acceleration, and use the fact that the resultant force is zero.
- If two bodies are involved, use the third law to identify equal forces

The following examples illustrate common ways of using the second law.

Showing the second law is equivalent to F = ma

The second law states that force is proportional to the rate of change of momentum.

Since momentum is given by mass × velocity, this means force is proportional to the rate of change of (mass × velocity).

If we assume mass is constant, then this becomes:

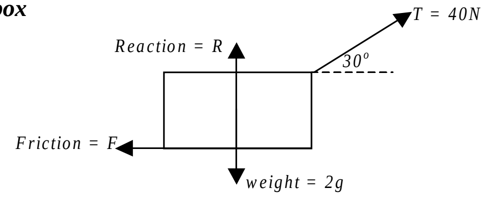
mass × (rate of change of velocity)

But rate of change of velocity is acceleration.

So we have force is proportional to mass × acceleration – or, as an equation, $F \propto ma$, or $F = kma$, where k is a constant of proportionality.

We get rid of the constant k by **defining** the **newton** to be such that a force of 1N gives a mass of 1kg an acceleration of $1ms^{-2}$

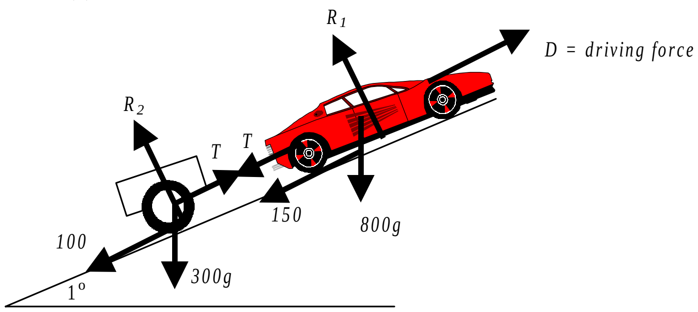
Example 3.A box of mass 2kg is being towed along a rough horizontal surface by a person pulling it on a string. The string is at 30º to the horizontal, and its tension is 10N. The box is accelerating at $1.0ms^{-2}$. Taking $g = 9.81ms^{-2}$, find:
(a) the frictional force acting on the box.
(b) the magnitude of the normal reaction force exerted by the ground on the box



(a) We need to resolve in the direction of the acceleration – which is horizontal, since the box is moving on horizontal ground:
$T\cos 30^o - F = ma = 2 \times 1$
So $F = 10\cos 30^o - 2 = 6.66N$ (3 SF)

(b) To find any other information, we need to resolve perpendicular to the direction of acceleration, and use the fact that resultant force is zero:
$T\sin 30^o + R - 2g = 0$
So $R = 2g - 10\sin 30^o = 9.62N$ (3SF)

Example 4
A car of mass 800kg is towing a trailer of mass 300kg up a road inclined at 1º to the horizontal. The car exerts a constant driving force, and starting from rest, achieves a speed of $10ms^{-1}$ in 50 seconds. The frictional forces on the car and trailer are constant, and of magnitude 150N and 100N respectively. Take $g = 10ms^{-2}$
Find: (a) the driving force of the car
(b) the tension in the tow bar.



(a) We first need to work out the acceleration from the information given. Since all the forces are constant, we know the acceleration will be constant, so constant acceleration equations can be used:
Using $v = u + at$: $10 = a \times 50 \Rightarrow a = 0.2ms^{-2}$

We now need to use F = ma. We must resolve in the direction of the acceleration – that is, up the hill

We can consider the car and trailer together – this will avoid bringing in the tension in the tow rope:
$D - 800g\sin 1^o - 300g\sin 1^o - 100 - 150 = (800 + 300)a = 1100 \times 0.2$
So $D = 1100 \times 0.2 + 800g\sin 1^o + 300g\sin 1^o + 100 + 150 = 662N$

> ***Tip:*** *You could consider the car and trailer seperately – you would then get two equations, which you would have to solve to find D*

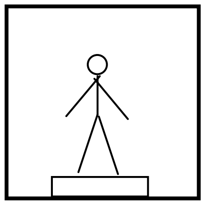
.(b) Since we need the tension here, we must look at either the car or the trailer – it doesn't matter which.
Trailer: $T - 100 - 300g\sin 1^o = 300 \times 0.2 \Rightarrow T = 212N$

> ***Tip: Many students lose marks by ignoring the weight of the car.***
> *If a hill is involved, the weight will come in to your equations!*

Example 5. A slimming club is situated at the top of a tall building; to motivate its clientele, the club has installed its own lift which contains a weighing machine. The lift accelerates uniformly at $1.0ms^{-2}$ for 90% of its journey, both going up and coming down.
Taking $g = 10ms^{-2}$, calculate:
(a) The resultant force required to accelerate a person whose mass is80kg at $1.0ms^{-2}$.
(b) The reading (in kg) on the weighing machine when the 80kg person stands on it as the lift accelerates upwards.
(c) The reading (in kg) of the machine when the same person stands on it as it accelerates downwards.

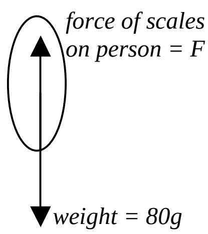
(a) We use F = ma: $F = 80 \times 1 = 80N$

(b) To work out the reading on the scales, we need to consider the forces acting on the ***person***



They are in contact with the scales, which are pushing them upwards. Their weight acts downwards.

So for the person, we have



We also know that the resultant force on the person is 80N upwards, from (a). This gives us:
$F - 80g = 80$

Now the reading on the scales is worked out from the force the person exerts on the scales.

The third law tells us that the force exerted by the person on the scales is the same in magnitude as the force exerted by the scales on the person – so it is F = 80g + 80 downwards.

The scales are calibrated to give a mass reading, rather than a "weight" reading.

So the mass reading on the scales will be obtained by dividing the force reading by g.

So the mass reading is $F \div g = (80g + 80) \div g = 88kg$

(c) When the lift is moving downwards, the resultant force is 80N downwards, so we have: $80g - F = 80$.

This gives $F = 80g - 80$*, and hence a mass reading of* $(80g - 80) \div g = 72kg$

Experimental investigation of Newton's Second Law

To investigate Newton's Second Law, a trolley on a track with ticker-tape is used.

First, the track is friction-compensated by tilting it until the trolley will run down at a steady speed – this occurs when the dots on the ticker-tape are evenly spaced.

A piece of elastic is attached to the trolley. A person pulls on the elastic so that it is always kept at the same length as the trolley moves - this provides a constant force on the trolley.

Different forces can be investigated by using two or three identical pieces of elastic. Different masses can be investigated by stacking trollies on top of each other.

To analyse the results, the ticker-tape is cut into 10 – dot lengths. Since 50 dots are produced every second, this allows the average velocity every 0.2 seconds to be calculated. From these velocities, the acceleration can be calculated.

The variation of the acceleration with applied force (number of pieces of elastic) and with mass (number of trollies) can then be analysed.

Questions

1. State Newton's second law, and explain how it leads to "F = ma"
2. Define the newton
3. Explain why the first law refers to "constant velocity", not "constant speed"
4. State three characteristics of a Newton's third law pair of forces.
5. Explain why, if you are leaning on a shut door and it suddenly opens, you fall over.
6. A car of mass 700kg is moving at a steady speed up a slope inclined at 2^o to the horizontal. The frictional resistance to motion is constant, and of magnitude 100N. Take $g = 10ms^{-2}$
   a) Calculate the driving force of the car

   The car now travels down the same slope, with the engine exerting the same driving force.
   b) Calculate the acceleration of the car.
7. A person of mass 60kg is standing in a lift of mass 250kg. The lift accelerates upwards at $1.5ms^{-2}$. Taking $g = 9.8ms^{-2}$, calculate
   a) the tension in the lift cable
   b) the force exerted on the person by the floor of the lift

Answers

1. –4. answers may be found in the text
5. The door was pushing back on you with a force equal in magnitude to the force you exerted on it. When the door is opened, this force is removed, so the forces on you are unbalanced, so you fall towards the door.
6. a) 344N b) $0.698ms^{-2}$ (both 3SF)
7. a) 3500 N (3SF) b) 678N

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

A manned rocket consists of a main rocket with spacecraft attached, with combined mass of 1.0×10^4kg, and a separable propulsion unit complete with its own first stage motor and fuel. The mass of this separate unit is 500 kg.

Both parts of the rocket contain motors capable of producing a constant thrust of 1.2×10^5N and are used in turn, the one in the main body igniting as soon as the separate unit has run out of fuel and been jettisoned. The rocket takes off from the ground and continues to fly vertically upwards. Take $g = 9.8ms^{-2}$

(a) Ignoring the effects of air resistance, calculate the:

(i) resultant force on the rocket at the instant of take-off. [2]

$1.2 \times 10^5 – (1\times 10^4 + 500)g$ ✓ = 15000N ✗ 1/2

> The student knows the correct calculation to carry out and has used the correct method, but has lost the final mark by using $g = 10ms^{-2}$ instead of $9.8ms^{-2}$ as it says. Read the question!

(ii) initial acceleration of the rocket. [2]

F = ma

$15000 = (1 \times 10^4 + 500)a$ ✓ so $a = 1.4286ms^{-2}$ ✗ 1/2

> Again, no difficulties understanding what is required, and the student could have gained full marks on this section from using the wrong answer to a) i) correctly, but the final mark is lost through using an inappropriate number of significant figures.

(b) Rocket fuel is burned at a steady rate of $2.5kg\ s^{-1}$. The first stage motor has 200kg of fuel available. Calculate the:

(i) time taken to use up the fuel in the first stage. [1]

$200 \div 2.5 = 80$ seconds ✓ 1/1

(ii) acceleration of the rocket at the instant just before the first stage fuel runs out.

$15000 = (1 \times 10^4 + 500 – 200)a$, ✓ so $a = 1.46ms^{-2}$ ✓ 2/2

> The student has gained full marks here, despite having a numerically incorrect answer, since the answer to a) i) has been used correctly and the answer is given to a suitable number of significant figures.

(c) The rocket motor in the main body ignites and begins to supply full thrust of 1.2×10^5N at the instant the first stage finishes and falls away. For the instant in time just after separation of the stages, calculate the:

(i) acceleration of the main rocket. [2]

$1.2 \times 10^5 - 1 \times 10^4 g = 1 \times 10^4 a$. ✓

$a = 11ms^{-2}$ ✗ 1/2

> Again, the student understood the calculation to be carried out, but has made a numerical error. Since s/he showed working, a mark can be awarded. The size of the answer – so different from earlier figures – should have alerted the student to the fact that the answer was wrong.

(ii) resultant force on an astronaut of mass 70kg aboard the spacecraft. [2]

$mg – ma = 80g – 80 \times 11 = -80N$???? 0/2

> Had the student drawn a diagram and worked out the resultant force carefully, s/he probably would not have written "mg – ma" instead of "mg + ma". The student clearly realises the negative answer is wrong, but should therefore have checked previous work.

Examiner's answers

(a) (i) $1.2 \times 10^5 - (1 \times 10^4 + 500) \times 9.8 = 17100$ N

(ii) $17100 = 10500\ a$ ✓ $1.69\ ms^{-2} = a$ ✓

(b) (i) 200/2.5 = 80 seconds ✓

(ii) $17100 = 10300\ a$ ✓ $a=1.7\ ms^{-2}$ ✓

(c) (i) $1.2 \times 10^5 - 10^4 \times 9.8 = 10^4 \times a$ ✓ $2.2\ ms^{-2} = a$ ✓

(ii) F = mg + ma ✓

= $70 \times 9.8 + 70 \times 2.2 = 840$ N ✓

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Physics Factsheet

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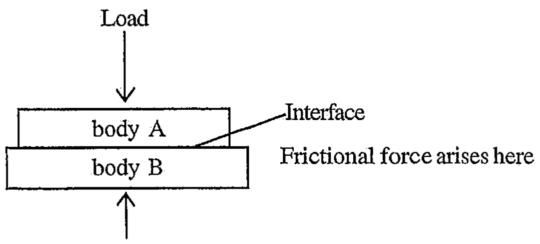
Understanding Frictional Forces

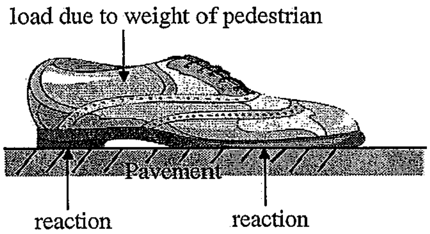
Significance of frictional forces in everyday life

Friction is a phenomenon which gives rise to forces which influence many activities in everyday life. Indeed, some activities depend on frictional forces, e.g. walking along a pavement, leaning a ladder against a wall, nailing two pieces of wood together. However, in other situations frictional forces are a burden to be overcome, e.g. energy loss in car engines.

Friction occurs at the interface between two bodies. No matter how smooth surfaces may appear, they are all "rough " on the microscopic scale. Frictional forces act along the plane of the interface.

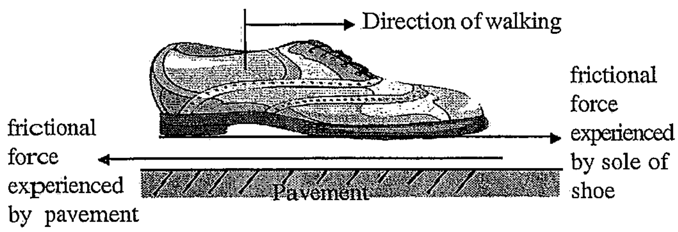


In the case of a person walking along a pavement, the interface is between the soles of the shoe and the pavement. The normal force is provided by the weight of the pedestrian.



The frictional force experienced by the shoe acts horizontally in the direction of walking. The force experienced by the pavement is in the opposite direction, but of equal magnitude *(Newton's Third Law of Motion.*

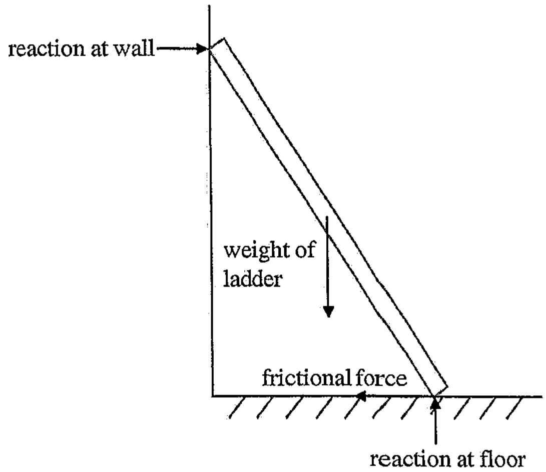
In these situations it is important to distinguish between the force exerted on a body, and that exerted by the body.



Walking depends on frictional forces. Indeed walking would be impossible if friction did not exist - try walking on ice.

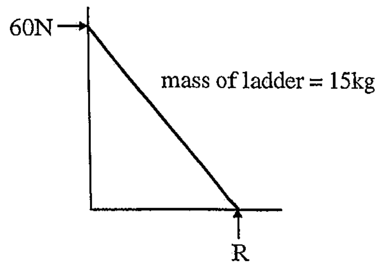
Notice that in the case of the shoe/pavement interface, frictional forces exist even when there is *no relative motion* between the two surfaces.

The simple act of leaning a ladder against a wall would be impossible without the existence of friction. An essential horizontal friction force acts at the interface between the floor and the bottom of the ladder, acting towards the wall. There may also be a friction force between the wall and ladder, but this is not essential.

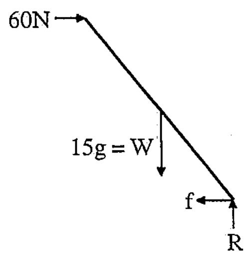


Exam Hint: Make sure you bring in the key points: there must be two bodies; frictional force acts along the interface; there may, or may not be, relative motion. Wording in exam questions, "rough" is code for ***include friction*** *"smooth" is code for* ***omit friction****.*

Example 1: A uniform ladder of mass 15kg rests with its upper end against a smooth vertical wall, and its lower end on a rough horizontal surface. If the magnitude of horizontal reaction at the wall is 60N, and g = 9.81ms^{-2}, then
(a) find the magnitude of the frictional force
(b) state its direction
(c) find the magnitude of the reaction R at the ground.



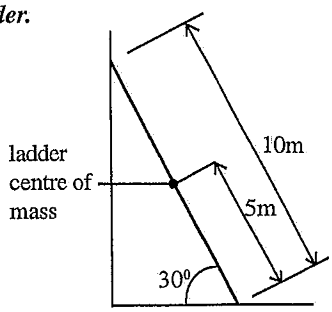
Answer:
Draw a free body diagram



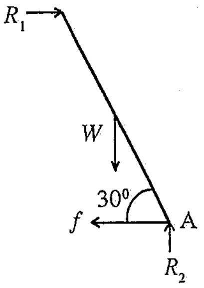
Horizontal forces: 60 = f
Vertical forces: R = W = 15g

(a) f = 60N
(b) towards the wall
(c) R = 15g = 147.1N

Example 2: The top end of a uniform ladder, length 10m and mass 20kg, rests against a smooth vertical wall at an angle of 30° to the horizontal as shown. Assume g = 9.81ms^{-2}.
Find the clockwise and anticlockwise moments about the bottom of the ladder.



Answer
Draw a free body diagram



Moments about A: clockwise:
$R_1 \times 10\sin 30 = 5R_1$ Nm.
Moments about A: anticlockwise:
$W \times 5\cos 30 = 196.2 \times 4.33 = 850$ Nm.

(These could be equated to find R_1, which is equal and opposite to the friction at the bottom of the ladder.)

If relative motion does exist at the interface between the two bodies, as for example in a car engine, then energy is expended. A force is exerted through a distance. This "friction overcoming" energy is transferred to another form, usually heat, which has to be removed by the cooling system.

Three possible requirements for frictional forces have been identified:
1. Minimise the friction, e.g. sliding friction in piston engines
2. Control the friction, e.g situations where it is required that the friction force remains within certain values, as in a clutch or friction drives.
3. Maximise the friction, e.g. brake pads/disc, tyre/road interface.

We cannot eliminate frictional forces entirely. Nonetheless, engineers and designers devote a great deal of effort in *reducing* unwanted energy losses due to friction:

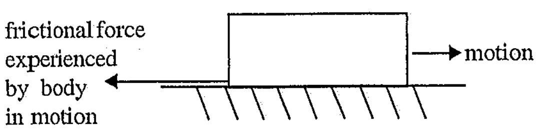
(a) Use low friction materials e.g. polymers
(b) Insert a rolling element into the interface e.g. ball bearings
(c) Use a lubricant between the two bodies e.g. oil.

Engineers *maximise* frictional forces by choosing materials with high friction e.g. in car brakes and clutches.

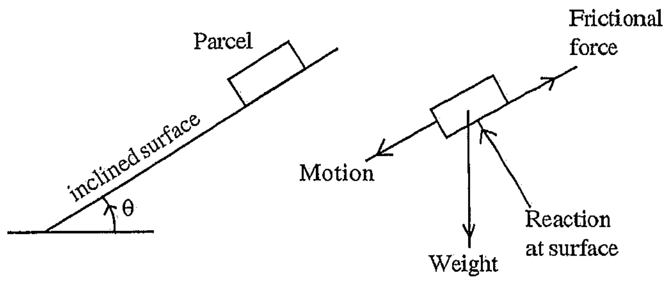
Frictional forces are essential for some everyday activities. In other activities, friction is a burden to be overcome

In which direction does the friction force act?

If one of the bodies that are in contact has motion relative to the other, then the frictional force always acts in a direction to oppose the motion.

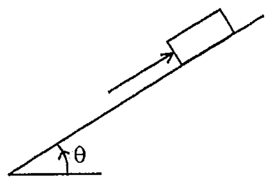


However, frictional forces can act when there is no relative motion between the two contacting bodies. In such cases it is necessary to examine each situation in order to decide the direction of the frictional force. Let us look at the case of a parcel on a slope inclined at an angle θ to the horizontal.



If the angle θ is sufficiently large then the parcel will slide down the slope, with the friction force experienced by the parcel acting up the slope, i.e. opposing motion.

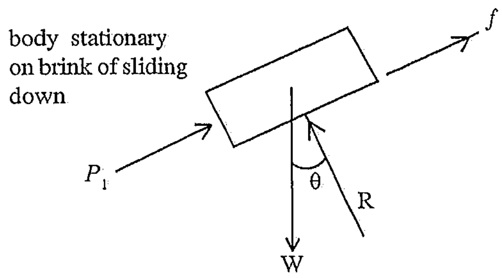
Suppose an additional force P is applied up the slope. If initially the force P is very small then the parcel will continue sliding (but more slowly) down the slope.



However force P can be increased until it just prevents the parcel from moving down and it remains stationary. If now the force P is further increased, it will reach a value at which the parcel is still stationary, but on the brink of sliding up the slope.

Now look in more detail at these two extreme cases where the parcel is stationary but on the brink of moving. To do so, it is necessary to draw Free Body Diagrams. These show all the external forces acting on a body

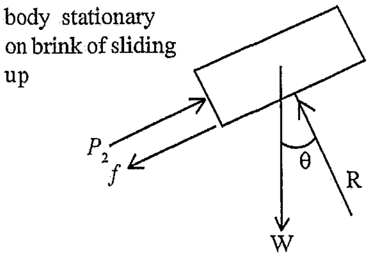
(a) Force P to just prevent the parcel sliding down the slope



Parcel is not moving, but on the brink of sliding down
Forces acting in the plane of the slope are:
(i) force P_1 up the slope
(ii) component of weight (W sin θ) down the slope
(iii) frictional force f acting up the slope

In this case the frictional force aids the force P in preventing sliding down. And then the other extreme situation:

(b) Force P is just less than that which will cause the parcel to move up the slope



Parcel is not moving but on the brink of sliding up.

Forces acting in the plane of the slope are;
(i) force P_2 up the slope
(ii) component of weight (W sinθ) down the slope,
(iii) frictional force f acting down the slope

In this case, friction acts down the slope, making it more difficult for the larger force P to move the body up the slope.

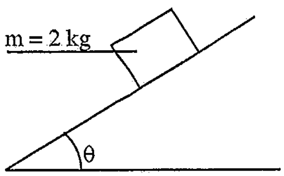
> ***Exam Hint:*** *Exam questions are often framed around this type of example. They may ask for the range of values a force P can take without the body moving.*

In the above cases the range of force is $(P_2 - P_1)$.

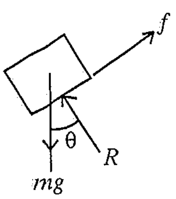
Analysing forces is much simpler if you draw a free body diagram. This comprises just the body together with **all** the forces experienced by the body. Use this free body diagram to write the equations of motion. Then solve the equations for the required quantity.

> ***Exam Hint:*** *Remember that a free body diagram should only show forces acting on on the body, not forces exerted by the body.*

Example 3: A body of mass 2kg is stationary on a rough surface which is inclined at θ° to the horizontal. The maximum frictional force between the body and the surface is 10N. The angle θ° is increased until the body is on the brink of sliding down. Calculate the value of θ at which this occurs. Take g as 9.81ms⁻².



Answer
Draw a free body diagram



body is on the brink of sliding down
friction acts up the slope
mg = weight of body
f = frictional force
R = reaction from surface

Forces parallel to slope: f = mgsinθ
$\sin\theta = f / mg = 10 / (2 \times 9.81) = 0.51$
so θ = 30.6°

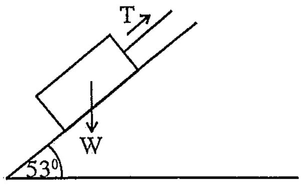
> *If motion exists, then the friction force acts in the opposite direction, i.e. to oppose the motion.*

> *Frictional forces can exist even when there is no relative motion between the two bodies*

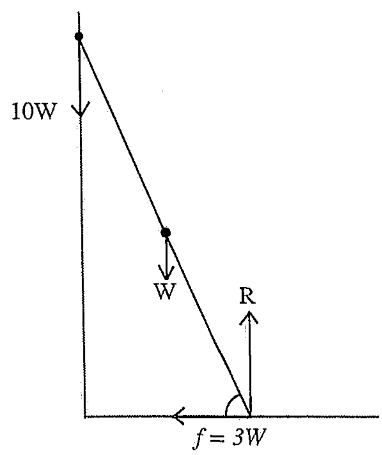
> *Always draw a free body diagram*

Practice Questions

1. A box of mass 5 kg is held at rest on a rough slope by a tensioned rope. The slope makes an angle of 53° with the horizontal. The frictional force at the interface between box and slope is 12 N.
   (a) Find the component of the weight acting down the slope. Take $g = 9.81\ \mathrm{ms^{-2}}$
   (b) Find the smallest value of T (the tension in the rope) which would stop the box from sliding down the slope. (Friction acts up the slope).
   (c) Find the largest value of T which could be applied without pulling the box up the slope. (Friction now acts down the slope.)



2. A ladder of weight W newtons leans against a smooth vertical wall and rests on a rough horizontal surface. A man weighing 10W newtons stands on the top of the ladder (just where it touches the wall). The frictional force at the ladder/ground interface is 3W newtons. Use moments about the top of the ladder (where the man is standing) to calculate the angle between the ladder and the ground. (Notice that in questions of this type any value can be used for the length of the ladder and for the weight, W.)



Answers

1. (a) 39.2N (b) $T_{min} = 27.2$ N. 9(c) $T_{max} = 51.2$ N
2. 74°

Acknowledgements:
This Physics Factsheet was researched and written by D. W. Parkins.
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Physics Factsheet

www.curriculum-press.co.uk Number 96

Dynamic Experiments Using Light Gates

Light gate set-ups almost always allow us to measure the time an object takes to travel a set distance. This, of course, allows us to calculate the average velocity over this distance.

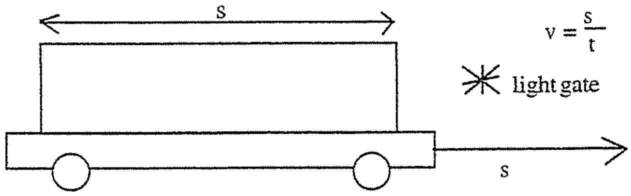
$v = s / t$ (in a stated direction)

Combinations of light gates can then be used in a myriad of investigations of velocity, acceleration, momentum and kinetic energy.

The basic calculation

(a) Single Light Gate:

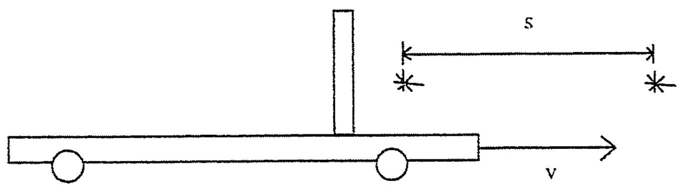
A card on a trolley switches a timer on, then off, as it passes in front of a light gate.



> *Example: The card is 20cm long and takes 0.2s to pass in front of the light gate. Find the average velocity of the trolley.*
>
> *Answer:*
> *Ave. velocity,* $v = s / t = 20 / 0.2 = 100 cms^{-1}$ *towards the right.*

(b) Twin Light Gates:

A thin vertical rod mounted on a trolley, passes through two light gates turning a timer on, then off.



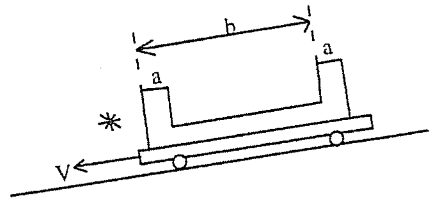
> *Example: The gates are separated by 50cm and the time recorded is 2.0s. Find the average speed of the trolley over this region.*
>
> *Answer:*
> *Ave. velocity,* $v = s / t = 50 / 2.0 = 25 cms^{-1}$ *towards the right.*

The advantage of the twin light gate system is that the greater distance should lead to greater accuracy in the calculation.

> *A greater distance will lead to greater accuracy in the average velocity calculated. But remember that this technique works out the* ***average*** *velocity. To determine* ***instantaneous*** *velocity, a very small distance must be used to ensure that the velocity does not change over the period of measurement.*

Acceleration

A very simple technique using one light gate can be used to find the acceleration of a trolley down a slope. (This investigation, and others, depends on the software of the electronics being set up to make and display the appropriate measurements.)



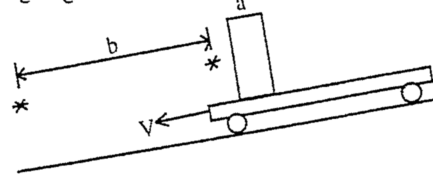
The gate is set up to record the 'on-off' time as each section 'a' passes the gate. The 'on-on' time as the front of each section 'a' blocks the beam is also recorded.

> ***Example:*** ***The following measurements are made.***
> ***a = 1.0cm, b = 18.0cm***
> ***time for first 'a' = 0.15s***
> ***time for second 'a' = 0.08s***
> ***time for 'b' = 1.88s***
> ***Find the acceleration of the trolley down the slope (assume uniform acceleration).***
>
> ***Answer:***
> $u = s / t = 1.0 / 0.15 = 6.67 cms^{-1}$
> $v = s / t = 1.0 / 0.08 = 12.5 cms^{-1}$
> $t = 1.88s$
> $a = (v-u) / t = 5.8 / 1.88 = 3.1 cms^{-2}$ *down the slope.*

> ***Exam Hint:*** *Remember that acceleration must have a direction (even if it is just "down the slope"). Velocity is also a vector. It is easy to forget this when looking at the results from the electronics, when the actual investigation may be out of sight and mind.*

With this set-up, the width of each section 'a' is significant compared to distance 'b'. We are not finding the instantaneous velocity at the front of each section, but the average velocity across each section 'a'. This could well have a measurable effect on the accuracy of the final calculation.

A more accurate method would use a larger distance 'b'. This would make the change in velocity over distance 'b' far greater than the change within each section 'a'. To manage this, it is necessary to use two light gates.

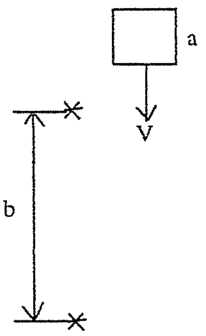


The calculation itself is the same. However you must ensure that your electronic system is set up to make the correct measurements.

Don't forget that factors such as friction can (and will) change as the velocity changes. Making 'b' too great can lead to a trolley changing acceleration significantly as it travels down the slope. This leads to further problems in the accuracy of the experiment, giving you an average value for acceleration with changing resistive force. This is probably not what you are attempting to find.

Free Fall

The acceleration of gravity (free fall) can be investigated in exactly the same way by dropping an object through two light gates.



The mathematics is exactly the same as in the previous situation. However getting the object to pass through both light beams is not necessarily trivial.

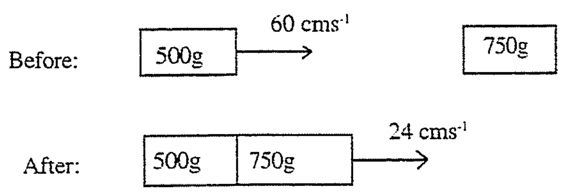
Momentum and Kinetic Energy in Collisions

The idea of conservation of momentum in collisions (and explosions) can be verified using light gates. And the loss of kinetic energy to heat and sound in collisions can also be investigated.

As we want constant velocity before and after the event, it is better to use air tracks rather than trolleys for collision experiments.

There are two main types of investigations – where the vehicles rebound freely from each other, and where they stick together after the collision. And within each of these, you can investigate collisions where both vehicles are moving before the collision, or more simply here one is stationary.

Example: On a horizontal air track, a 500g vehicle collides with and sticks to a stationary vehicle of mass 750g. Light gates allow us to measure the initial velocity of the 500g vehicle as 60cms^{-1} towards the right, and the final velocity of the pair as 24cms^{-1} towards the right. (See the diagram.) Find:
(a) the initial momentum of the 500g vehicle
(b) the final momentum of the pair
(c) the kinetic energy lost in the collision.

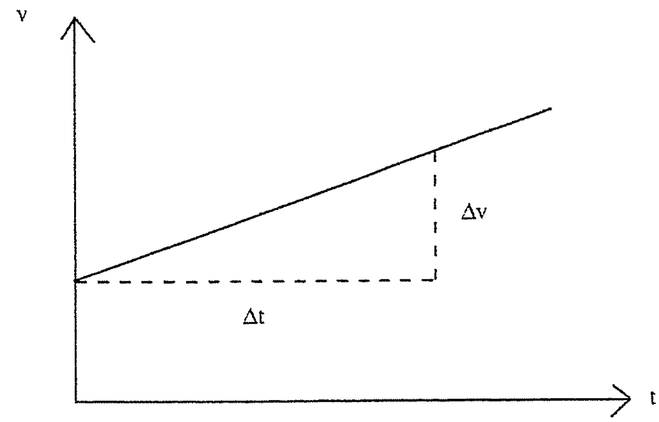


Answer:
(a) momentum = mv = 0.5 × 60 = 30kgcms^{-1} towards the right.
(b) momentum = mv = 1.25 × 24 = 30kgcms^{-1} towards the right.
(c) KE before = ½mv^2 = ½ × 0.5 × 0.60^2 = 0.090J.
KE after = 0.036J
The kinetic energy lost in collision is 0.054J. It is converted to heat and sound energy.

Although momentum is always conserved in a collision, some kinetic energy is almost always lost to heat and sound. Only in a completely **elastic** collision is KE conserved.

Use of Graphs

The values obtained for velocity and time can be converted to acceleration by determining the gradient of a **v-t** graph.

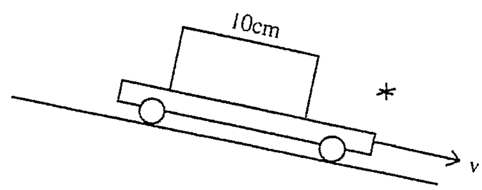


But remember that acceleration is a vector. When working from a graph, it is easy to forget to state the direction in your answer.

Questions

1. State an advantage and a disadvantage of using the two light gate system, rather than a single light gate, in determining average velocity.

2. A light gate is used to find the speed of a trolley as it rolls down a slope. The light beam is broken for 0.45s.



(a) Find the speed of the trolley as it passes the light gate.

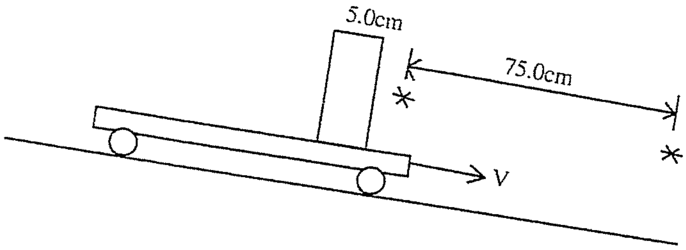
(b) State an advantage of shortening the card to 2cm rather than 10cm.

(c) State a disadvantage of doing this.

3. (a) Suggest three reasons why a light gate system could not be used to find the velocity of a high-speed neutron.

(b) Could such a system be used in determining the speed of a bullet? (Assume that the timing device measures to +/- one millisecond, and that the light gates could be placed 5 metres apart.)

4. This set-up is used to find the acceleration of a trolley down a slope.



Here is a table of the readings obtained.

Time to pass first light gate	0.60s
Time to pass second light gate	0.30s
Time between light gates	7.50s

(a) Find the velocity at each gate.
(b) Find the acceleration down the slope.
(c) Use an equation of motion to **calculate** the distance between the gates.
(d) Why is this different from the 75cm given in the diagram?

5. Here is the arrangement used for a conservation of momentum experiment on an air track. The two vehicles rebound from each other after the collision.

(a) How many times would each light gate be triggered?
(b) What other measurements, besides the time, would have to be made before conservation of momentum in a collision could be confirmed?

Answers:

1. An advantage is that the greater distance provides greater accuracy in the calculation. The only obvious disadvantage is that the twin light gate system is more complex to set up.

2. **(a) $v = s / t = 10 / 0.45 = 22cms^{-1}$**
(b) The speed is changing as the trolley accelerates. The shorter card would give a better estimate of instantaneous speed.
(c) A shorter card would reduce the time measured. This increases the percentage error in the calculation.

3. (a) The neutron is too small to break the beam, its velocity is so great that the system could not measure the time involved, and we have no means of aiming the beam along the correct path through the gates.
(b) A time of one millisecond over a distance of 5 metres would imply a speed of $5000ms^{-1}$. As long as the speed of the bullet was well below this, the system should be able to provide a result. Obviously the physical set-up would have to be very precise to break both beams.

4. (a) $u = 5.0 / 0.60 = 8.3cms^{-1}$ down the slope.
$v = 16.7cms^{-1}$ down the slope.
(b) $a = (v-u) / t = 8.4 / 7.50 = 1.12cms^{-2}$ down the slope.
(c) $v^2 = u^2 + 2as$, $s = (v^2 - u^2) / 2a = 94cm$.
(d) The acceleration cannot be uniform (as required by the equations of motion).

5. (a) Four times each – on, then off, before and after the collision.
(b) The mass of each vehicle and the length of the card on each vehicle.

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Physics Factsheet

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Why Students Lose Marks: AS Forces

This Factsheet analyses students' answers to AS exam questions on forces. By the end of this Factsheet, you should be more confident about:
- What the examiners want
- The kinds of things you are likely to be asked
- Common mistakes and misunderstandings

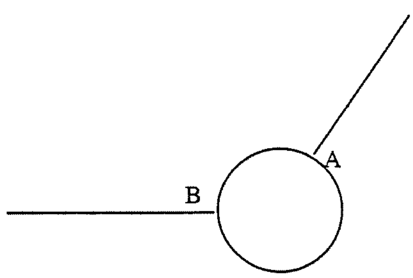
As you read the students' answers to the questions and the comments, try to work out what the student should have done - using the hints and comments if necessary - before you read the markscheme. NB: this Factsheet is **not** intended to teach you about forces for the first time - this is covered in Factsheet 12. Future Factsheets will cover the use of Newton's second law, momentum, work and power.

What do you have to know?

In this type of question, the examiner is trying to assess whether you:
- understand what a force is and are familiar with the standard types of force;
- can understand, draw and use free-body force diagrams;
- can find the resultant force
- can resolve forces into their components
- can find the moment of a force
- can use Newton's first and third laws

In addition, questions on forces require you to be confident with **vectors**, since forces are vector quantities. In any question involving **calculation**, there are likely to be marks available for showing a clear **method.** If you have to use one answer in the next part of the question, there are likely to be "error carried forward" marks available - so you are only penalised once for a wrong answer. Also remember you must always quote your answer to an appropriate number of significant figures.

The diagram shows a 200 g ball which is part of a child's game. The ball is suspended from a light piece of string at point A. The child pulls the ball to the side using a second light piece of string at point B. The tension in the horizontal string is 0.96N. The ball is at equilibrium.



(a) What balances the weight of the ball? *The tension* × *Not specific enough. Firstly, only one of the two tensions is involved, secondly, only one component of this tension balances the weight.* [1]

(b) Determine

(i) the magnitude of the horizontal component of the tension in the string at A

= tension in B = 0.96N ✓ *Correct - since the student is using the idea that horizontal forces balance and vertical forces balance, s/he should have realised that the answer to (a) was not specific enough and gone back to change it.* [1]

(ii) the magnitude of the vertical component of the tension in the string at A

= weight = 0.2 N × *The vertical component IS equal to the weight - but the weight is not 0.2 N ! This is a classic mistake - confusing mass and weight. Weight = mass × acceleration due to gravity.* [1]

(iii) the angle the string at A makes with the vertical

tanx = 0.96/0.2 ✓✓ *The student is given "error carried forward" marks here from the incorrect answer in (ii)*

x = 78° [2]

Markscheme

(a) the vertical component of the tension in the string at A (1)

(b) (i) 0.96N (1)

(ii) 0.2kg = 0.2 × 9.81 = 1.962 N (1) (allow 2.0 or 1.96)

(iii) tanθ = 0.96/1.962 (1) θ = 26°

An object is acted upon by two forces at right angles to each other. One of the forces has a magnitude of 6.0N and the resultant force produced on the object is 9.8N. Determine

(i) the magnitude of the other force

$\sqrt{(9.8^2 - 6.0^2)} = 7.7485$

✓× The student clearly understands how to do this question - s/he is using Pythagoras' theorem, with the resultant force as the longest side. S/he has lost the second mark because of the inappropriate degree of accuracy - the data in the question are given to two significant figures, so why give the answer to five? The units have also been omitted. It is generally NOT a good idea to try to do this sort of question without a diagram - even if the question doesn't tell you to draw one.

(ii) the angle between the resultant force and the 6.0N force

sinx = 6.0/9.8

x = 38°

×× No marks here as the student has used sin when cos should have been used - with a diagram, this mistake probably would not have happened.

[4]

Markscheme

(i) $F^2 + 6.0^2 = 9.8^2$ (1) F = 7.7N (1)

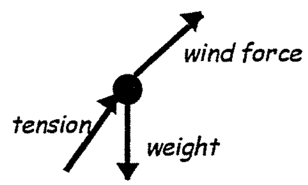
(ii) $\cos\theta = 6.0/9.8$ (1) $\theta = 52°$ (1)

A child is holding a string attached to a flying kite. The weight of the kite is 3N. The wind exerts a force on the kite, in the direction upwards and away from the child. The kite is held in equilibrium.

(a) State the **two** other forces acting on the kite *weight, tension* [2]

✓✓ Both marks awarded here - but it would have been better for the student to say "weight of kite" and "tension in the string" to make it absolutely clear.

(b) Draw a free-body force diagram to show the forces acting on the kite.



✓✓× The weight is correct - acting vertically downwards.
The wind force gets the mark (although to be really accurate, it should be nearer the vertical than the tension is)
*No marks for the tension - the student appears to think it is **pushing** the kite rather than pulling it!*
This diagram clearly can't be correct, as there would be a resultant force to the right - ie the kite would not be in equilibrium. [3]

(c) The angle between the string and the vertical is 36°. The tension in the string is 30N.

(i) Find the magnitude of the horizontal component of the force due to the wind

= horizontal component of tension

= 30sin36

= 17N (2 SF)

✓✓× The method is good - it was sensible of the student to put in their first line, since this makes it clear s/he is using the fact the kite is in equilibrium.
The calculation method is correct - the final mark is lost because of incorrect rounding - the answer on the calculator (17.6) should be rounded to 18N.
It is sensible to round to 2 significant figures, as the data is given no more accurately than this. [3]

(ii) Find the magnitude of the vertical component of the force due to the wind

vertical wind + vertical tension = weight

W + 30cos36 = 3

W = -21N so 21N

×✓× The student is using the wrong idea that tension is pushing again (why? S/he did not use this wrong idea in part (i)). This means that the first line is incorrect. A mark is given for resolving correctly. The student should have realised a negative answer was not correct - this would mean the wind was acting downwards and away from the child, rather than upwards and away as stated in the question. [3]

(iii) Hence calculate the magnitude of the force due to the wind.

$21^2 + 17^2 = 730N$

✓× The student starts out with a correct method - using Pythagoras' theorem. But then s/he makes a very silly mistake - forgetting to square root the answer. Note follow-through marks are given for the use of incorrect answer from (ii)
If the student had done a "plausibility check" on the answer, s/he would realise that a figure in the hundreds was unlikely, given the size of the forces in the rest of the question. [2]

Markscheme

(a) weight of kite (1) tension in string (1)

(b) weight acting vertically down (1) tension acting down at an angle (1) wind force acting up at an angle, on opposite side to tension (1)

(c) (i) same as horizontal component of tension (1) *(can be implied)* 30sin36 (1) 18N (1)

(ii) weight + vertical component of tension (1) 30cos36 (1) + 3 = 27N (1)

(iii) $18^2 + 27^2 = F^2$ (1) F = 32N (1)

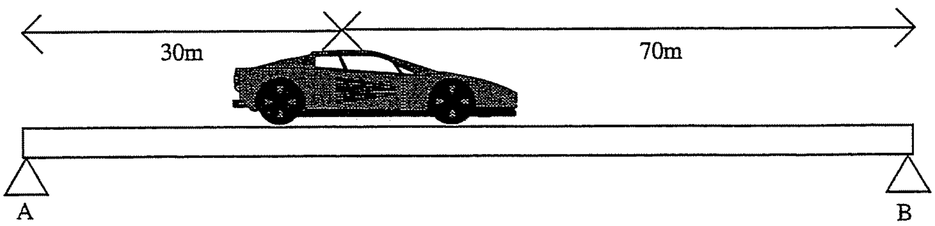
(a) State the principle of moments

Total clockwise moments = total anticlockwise moments

×✓ *The student omitted "for a body at equilibrium" - this is very important, since the principle is only true for a body at equilibrium! S/he also did not state that the moments had to be taken about the same point; no marks would be lost here for this, as only two marks were available, but it might be required elsewhere.*

[2]

(b) The diagram shows a car of weight 6000N at rest on a bridge of weight 8000N. The bridge rests on supports at A and B. The centre of mass of the car is 30 m from A and 70m from B. The centre of mass of the bridge is at its centre.



(i) State the **total** of the two upward forces at A and B, and explain how you obtained your answer

14000N because the forces must balance

✓× *The student understands the idea, but is not being precise enough - s/he should have mentioned that the upwards and downwards forces balance because the car is in equilibrium.*

[2]

(ii) Find the force acting at support A

Clockwise moments about B: 100×A

Anticlockwise moments about B: 70 ×6000 + 50 ×8000

A = (420000 + 400000)/100 = 8200N

✓✓✓× *The calculation is correctly done and well set-out. The method is sensible - taking moments about B avoids involving the force at B. The final mark is not gained because the question asked for the* ***force****, not just the* ***magnitude*** *of the force - so its direction must be specified also.*

[4]

(iii) State the force exerted by the bridge on support B, and explain how you obtained your answer

Force = 14000 - 8200 = 5800N

✓× *The answer is correct, but no explanation for why this is the force* ***of the bridge on B****, rather than the force of B on the bridge (they are equal, but the student has not said why)*

[2]

Markscheme

(a) For a body in a state of equilibrium (1), total clockwise moments about a point = total anticlockwise moments (about the same point). (1)

(b) (i) 14000N (1) because as the car is in equilibrium total upward force = total downward force (1)

(ii) 100A = (70)(6000) + (50)(8000) (1 each side)
A = 8200 (1)
Force is 8200N upwards (1)

(iii) 1400 - 8200 = 5800N = force of support B on bridge (1)
Force of bridge on support is equal (Newton's 3rd law) (1)

Practice Question

1. A speedboat is towing a paraglider at a constant speed and height on the end of a light rope of length 30m, which makes an angle θ with the horizontal. The forces acting on the paraglider are the vertical lift, L, the horizontal drag, D, his weight, W and the tension in the rope, T.
(a) Draw a free body force diagram for the paraglider [2]
(b) State and explain the value of the resultant of these forces. [2]
(c) Hence, write an equation relating the magnitudes of:
(i) D, T and θ [1]
(ii) L, W, T and θ. [1]

2. A see-saw is made of a uniform beam of length 3m pivoted at its central point.
(a) Explain the significance of the fact the beam is uniform [2]
(b) Andy, who weighs 450N sits on one end of the see-saw. Beth, who weighs 200N sits on the other side, 0.5m from the pivot. Carl sits at the end of the see-saw on Beth's side. How much does he weigh if the see-saw is in equilibrium? [3]

Markscheme

1. (a) Lift vertically upwards, Weight vertically downwards, Drag horizontal, Tension diagonally downwards, opposite side to drag (½ each)
(b) zero (1) because paraglider is in equilibrium (const speed & height)(1)
(c) (i) D = T cosθ (1)
(ii) L = W + T sinθ (1)

2. (a) Centre of mass is at the midpoint (1) so no moment about pivot(1)
(b) 450 × 1.5 = 200 × 0.5 + W × 1.5 (1 each side)
W = 383N (1)

Acknowledgements: This Factsheet was researched and written by Cath Brown. ***Curriculum Press, Bank House, 105 King Street, Wellington, Shropshire, TF1 1NU. PhysicsFactsheets*** *may be copied free of charge by teaching staff or students, provided that their school is a registered subscriber. No part of these Factsheets may be reproduced, stored in a retrieval system, or transmitted, in any other form or by any other means, without the prior permission of the publisher. ISSN 1351-5136*

Physics Factsheet

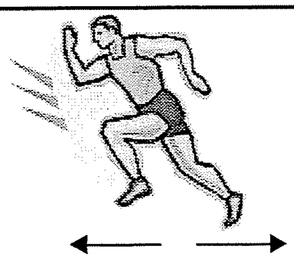
www.curriculum-press.co.uk Number 108

Where Students Make Mistakes: Balanced Forces and Action-Reaction pairs

Newton's third law of motion states, "For every action, there is an equal and opposite reaction." This means forces always act in pairs. In an exam question, the tricky part can be working out just what the pair is.

For every action, there is an equal and opposite reaction (forces act in pairs)

To start with, we'll use several simple examples to help you understand Newton's third law.



Force of earth acting on the sprinter ← → Force of sprinter acting on the earth

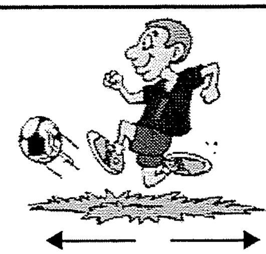
What happens when a sprinter begins their race? They push off from their starting blocks. If the sprinter is pushing backwards against the earth, what's the other half of the force pair? The earth must be pushing the sprinter forwards, with a force of exactly the same size. But if the forces are equally sized and in opposite directions, how can the sprinter move?

The sprinter moves because only one of the forces acts on him. The other acts on the Earth.

The sprinter pushes the earth backwards and the earth pushes the sprinter forwards. But the sprinter is tiny compared to the earth. We notice the sprinter moving forwards but we don't see the earth moving backwards.

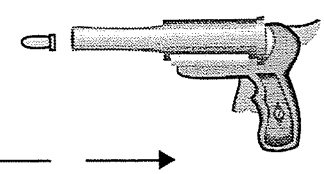
Exam Hint: *Even though force-pairs are always equal and opposite, always balanced, there can still be movement. As we walk, we are pushed forwards and the earth is pushed backwards. Because the earth is so huge, this backward motion is tiny and we don't see it.*

A footballer kicks a ball forwards. What is the other half of the force pair? The ball pushes the foot backwards. If we kick the ball forwards hard, the backward force acting on the foot could break a toe!



Force of foot acting on the ball ← → Force of ball acting on the foot

What about a bullet fired from a gun? The gun forces the bullet forwards. The bullet forces the gun backwards. Our hand or shoulder feels a kick from this "recoil" (although there are mechanical ways to deal with this).



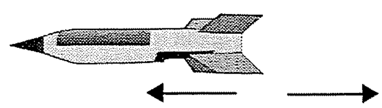
Force of gun acting on the bullet ← → Force of bullet acting on the gun

Example Exam Question: An Olympic pistol marksman fires at a target. The 10g bullet accelerates at $2\times10^6 ms^{-2}$. Assume his gun has a mass of 2kg. Explain and calculate the acceleration the gun undergoes.

Answer

Action of the gun on the bullet has an equal and opposite reaction of the bullet on the gun. Gun's mass is 200 times greater than the bullet, so the gun's acceleration will be 200 times less.

$$a = \frac{2\times10^6 ms^{-2}}{200} = 1\times10^4 ms^{-2}$$

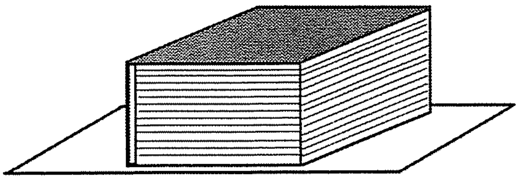


Force of exhaust gases acting on the rocket ← → Force of rocket acting on the exhaust gases

How does a rocket move? Many people think it takes off by pushing against the ground. So how can it work in space? A rocket engine produces high speed exhaust gases which are forced backwards by the rocket. The exhaust gases push the rocket forwards.

> ***Exam Hint:*** *A rocket does NOT travel by pushing off against the ground. The exhaust gases moving backwards push the rocket forwards.*

Now, what is the force pair in this picture?

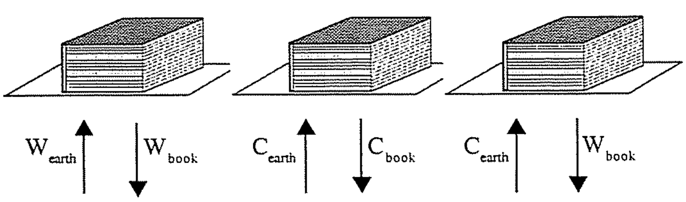


You may be tempted to say the weight of the book is balanced by the contact force from the ground. Although this IS true, there are TWO separate force pairs here. We need to understand both pairs before going further.

Gravity is an attraction between any objects with mass. The book and the earth are attracting each other. For a small object, we usually think only about the force of the earth pulling the book DOWNWARDS, causing weight. But the book is also attracting the earth UPWARDS, with a force of exactly the same size (but in the opposite direction). Because the earth is so much larger than the book, we don't notice the effect of the book on the earth.

> Weight acts in a force pair. An object is attracted down to the earth and so the object attracts the earth upwards. The forces are equal in size, but in opposite directions.

The book and the ground are also pushing against each other. These are contact forces. Everyday matter consists of atoms. An atom consists of a positive nucleus surrounded by shells of negative electrons. A simple way to consider contact forces is that if you bring two surfaces close together, the outer electrons repel each other. The reason a book does not fall through a table is because the outer electrons in the book and the table repel each other. In reality, they're not actually touching. The electrostatic repulsion causes the book to "float" above the table.



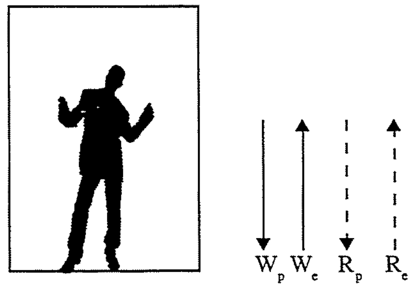
1) weight force pair 2) contact force pair 3) forces acting on book

> Contact forces act in a force pair. The outer electrons in two surfaces repel each other.

> ***Exam Hint:*** *The weight of an object and the upward contact force are not a force pair. The earth-object attraction is one force pair and the contact force pair is caused by the two surfaces repelling each other.*

Now, what happens if an object is moved up or down?

Think about travelling in a lift. In the first example, the lift is not moving.



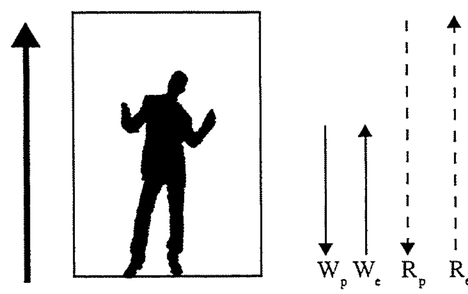
Stationary lift

According to Newton's first law of motion, something will stay still unless an unbalanced force acts. In this case, the man's weight (W_p) is balanced by the upwards contact force (R_e). He stays where he is. However, Newton's first law goes on to say that something will stay at a steady speed in a straight line unless an unbalanced force acts. In the first example, the lift could be moving up or down at a steady speed.

> When an object on a surface is at rest or moving vertically at a **steady** speed, the upward contact force is equal and opposite to the weight of the object.

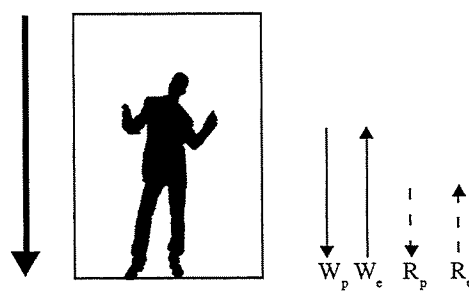
If the lift **accelerates** upwards or downwards, the weight of the man due to the earth (W_p) and the earth due to the man (W_e) DO NOT CHANGE.

In the second example, the man is being forced upwards. The upwards contact force must be bigger than his weight downwards. This overall unbalanced upward force causes upward acceleration. In more simple terms the floor is pushing him upwards.



Lift accelerates upwards

In the third example, the lift is moving down. The upward push due to contact force is less. The overall unbalanced force is downwards and he accelerates downwards. In more simple terms, the floor is "falling away" below him.

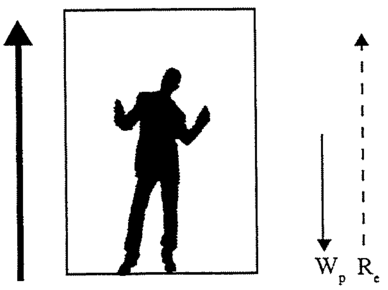


Lift accelerates downwards

> ***Exam Hint:*** *When an object is resting on a surface, the upward contact force will reduce if the surface accelerates downwards and increase if the surface accelerates upwards.*

How does the upward contact force alter during exercise, say, if a woman is repeatedly squatting and standing? In the initial part of the squat, the woman accelerates downwards. Is the contact force greater or less than her weight? The overall unbalanced force must be downwards, which means the upward contact force is SMALLER. As she reaches the bottom of the squat, she decelerates to a stop. The overall unbalanced force must be upwards. The upwards contact force must be BIGGER than her weight. If she pauses here, the upward contact force will BALANCE her weight again. As she rises, she accelerates upwards, so upward contact force is GREATER. As she decelerates to a stop again, the upward contact force must be LESS.

Forces acting on <u>man</u> accelerating upwards



Try this for yourself on a set of scales. You need to explain why the needle wobbles as you squat and rise.

Questions

1) Explain Newton's third law of motion in your own words.

2) Use Newton's third law of motion to explain why it is difficult to walk on a slippery surface.

3) Explain and calculate the acceleration of a 5kg rifle when a 20g bullet is fired with an acceleration of $5 \times 10^6 ms^{-2}$.

4) Use a vector diagram to show the forces involved in striking a cricket ball with a bat.

5) In your own words, explain how a rocket moves.

6) Explain the two forces pairs acting on an object resting on the ground.

7) Describe and explain how the pointer on a set of bathroom scales would change if you squatted and then stood.

Answers

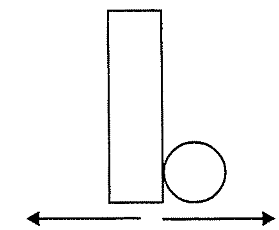
1) Newton's third law of motion states that for every action, there is an equal and opposite reaction. This means forces always act in pairs, equal in size and opposite in direction.

2) When we walk, we push back against the earth. According to Newton's third law of motion, the earth pushes us with the same force in the opposite direction. If we remove friction, then we reduce our ability to push backwards and then be pushed forward.

3) The gun forces the bullet forwards so the bullet forces the gun backwards. The mass of the gun is 250 times greater than the bullet, so the acceleration is 250 times less.

$$a = \frac{5 \times 10^6 ms^{-2}}{250} = 2 \times 10^4 ms^{-2}$$

4)



5) A rocket engine produces exhaust gases which are forced out at a high velocity. The rocket forces the exhaust gases backwards and the exhaust gases force the rocket forwards.

6) The two force pairs acting on an object resting on the ground are weight and contact force. Objects with mass attract each other by gravity, causing weight. The weight of the object caused by the earth is exactly equal and opposite to the weight of the earth caused by the object. Contact forces occur because the outer electrons in the atoms of the object and the ground repel each other via electrostatic repulsion.

7) Let's say the scales read 50kg (really, they're measuring 500N weight but the scale is written in kg). As you accelerate downwards, the upward contact force is reduced. The scales would dip below 50kg (although your mass and weight ISN'T CHANGING!). Then as you decelerate to a stop, the upward contact force is greater; the pointer would increase above 50kg. When paused, the pointer would be at 50kg, as the upward contact force and your weight balance. As you accelerate upwards, the upward contact force is greater and the pointer goes up. Decelerating to a finish causes the upward contact force to decrease and the scales would go below 50 kg. When stopped, the pointer reads 50kg again as your weight is balanced by the upward contact force.

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Physics Factsheet

September 2000 **Number 05**

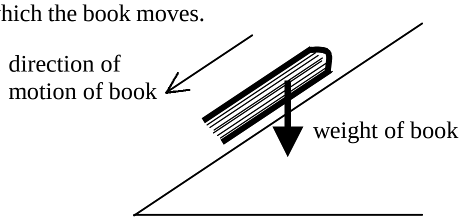
Work Energy & Power

1. Work

If a force acts on a body and causes it to move, then the force is doing **work**.

> ***W = Fs***
> *W = work done (J)*
> *F = force applied (N)*
> *s = distance moved* ***in the direction of the force*** *(m)*
>
> *This gives rise to the definition of the* ***joule****:*
> ***one joule of work is done when a force of 1 newton moves a body***

The above equation only works for a **constant** force; see the box for what happens with non-constant forces. Sometimes, the force causes something to move in a direction that's not the same as the direction of the force. The diagram below shows a book lying on a tilted smooth surface. If the surface is smooth enough, the book will slide down. The force causing the book to do this is the book's **weight**. The weight of the book does not act in the direction in which the book moves.



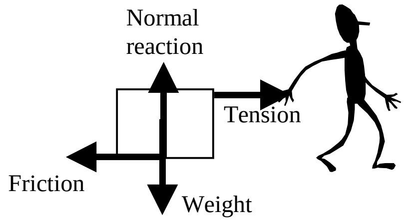
In this case, the equation is:

> *Work done by a force = distance moved × component of force in that direction*
>
> *Or, to save time resolving:* ***Work done = Fs cosθ****,*
> *where θ is the angle between the direction of the force and the direction of motion.*

Work is a **scalar** quantity, since it does not depend on the actual directions of the force and the distance moved, but on their magnitudes and the angle between them.

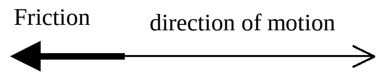
What happens if the object moves at right angles to the force, or in the opposite direction?

If you are dragging a box along the floor using a rope, then there are other forces on the box besides you pulling it:



No work is done by the weight and normal reaction forces, because they are **perpendicular** to the direction of motion. This ties in with the equation W = Fs cosθ, since in this case θ is 90°, and cos90° = 0.

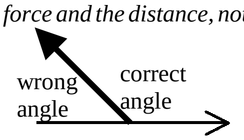
The friction is "trying" to stop the motion of the box. This means it does a **negative amount of work** on the box. (it can also be said that **the box does work <u>against</u> friction**). How does this tie in with the equation W = Fscosθ ?



Because the Friction force and the direction of motion are pointing in opposite directions, the angle between them is 180°.

Since cos180° = -1, this gives a negative value for work done.

> ***Tip:*** *When you are using the equation W = Fs cosθ, make sure θ is the angle between the directions of the force and the distance, not just between the lines representing them.*



What happens if the force is not constant?

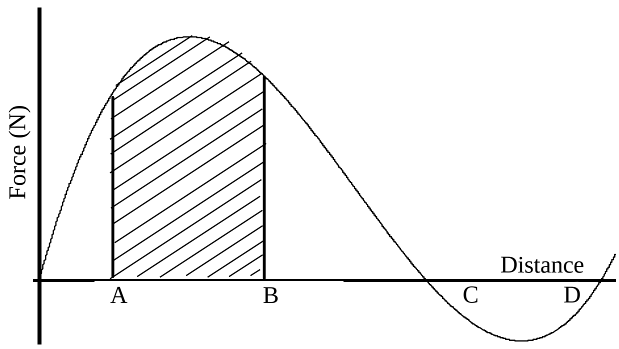
To find the work done by a non-constant force, there are two options:

- use the average force in the normal equation
- obtain the work done from a graph

To find out the work done from a graph, you draw a graph of force (on the y-axis) against distance (on the x-axis). The area **under** the graph is the work done.

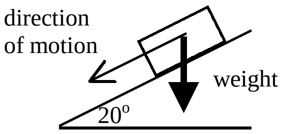
In the graph shown right, the shaded area gives the work done by the force in moving from point A to point B.

Note that between points C and D, the force becomes negative – meaning it is in the opposite direction – so the work done by the force will be negative (or equivalently, work will be done against the force).



Example 1. A smooth plank of wood has one end fixed to the floor and the other resting on a table, so that the wood makes an angle of 20° with the horizontal floor.

A book of mass 0.9kg is held so that it rests on the plank. It slides down 40cm before coming to rest at the bottom of the plank. Find the work done by the book's weight. Take g = 10ms^{-2}



To apply the formula W = Fs cosθ, we need to know the angle between the direction of motion and the force. By using angles in a triangle sum to 180°, we find it is 70°.

So W = 0.9 × 10 × 0.4 × cos 70° = 1.23J

(Alternatively, we could have found the component of the weight in the direction of the motion, which would be 0.9 × 10 × sin20°, then multiplied it by the distance. This gives the same result.)

2. Energy

Energy is the ability to do work

There are many different forms of energy – chemical, electrical, heat, light, sound, nuclear, kinetic and potential. In fact, almost all of these are types of **kinetic energy** or **potential energy**.

Kinetic energy
- *is the energy a body possesses because it is moving.*
- *It can be defined as the work required to increase its velocity from zero to its current value.*
- *k.e. = ½ mv^2, where m = mass (kg) and v = speed (ms^{-1})*

Potential energy
- *is the energy a body posseses due to its position (or the arrangement of its parts).*

Gravitational potential energy
- *can be defined as the work required to raise the body to the height it is at.*
- *g.p.e. = mgh, where,*
  m = mass (kg)
  g = acceleration due to gravity (ms^{-2}),
  h = height of centre of mass above some set reference level.
- *Note*
  - *it is arbitrary where we measure the height from – we can choose whatever level is convenient*
  - *if a body is **below** this level, then h, and hence its gravitational potential energy, are negative.*

***Tip**. You may be asked to describe the energy changes involved in a process. Remember:*
- *if there is any **friction** involved, some **heat energy** will be produced*
- *if the process makes a noise, **sound energy** will be produced*
- ***chemical energy** includes the energy inside batteries and energy gained from food*
- *if something is falling, it will be losing **potential energy***
- *if something is slowing down or speeding up, it will be losing orgaining **kinetic energy***

Typical Exam Question
(a) Is work a vector or a scalar quantity? Explain [2]
(b) A child of mass 26 kg starting from rest, slides down a playground slide. What energy changes occur during the descent? [3]

(a) Work is a scalar ✓ It is independent of the directions of the force or distance moved, and just depends on their magnitude.✓
(b) Gravitational potential energy ✓ changes to kinetic energy,✓ heat ✓

Deriving the formulae for kinetic and potential energy

Kinetic energy
Let us consider a constant force F acting on a body of mass m, so that it moves a distance s and increases its speed from 0 to v.

Then we know that the work done by the force is given by:
W = Fs
We also know, from Newton's 2nd Law, that
F = ma
where a is the acceleration of the body.

Putting these two equations together, we get
W = ma × s (1)

Now, from equations of motion, we know
2as = v^2 – 0^2 (since u = 0)
Rearranging this equation, we get
as = ½ v^2 (2)

Putting equation (2) into equation (1), we get
W = mas = m × ½ v^2 = ½ mv^2

Gravitational potential energy
Let us consider raising a body of mass m through a vertical height h.

The weight of the body is mg, acting downwards. To overcome this, a force of equal magnitude but opposite direction has to be exerted – in other words, a force of mg upwards.

The work done by this force = force × distance moved = mgh.

3. Conservation of energy

*The **principal of conservation of energy** states that:*
Energy may be transformed from one form into another, but it cannot be created or destroyed.

Here, we shall be considering a special case of this, which concerns **mechanical energy** – which is gravitational potential energy and kinetic energy.

*The **principal of conservation of mechanical energy** states:*
*The total amount of mechanical energy (k.e. + g.p.e.) which the bodies in a system possess is constant,**provided no external forces act**.*

The "no external forces acting" means that:
- there is no friction
- no other "extra" force – such as a car engine or someone pushing – is involved

Weight does NOT count as an external force.

The principal of conservation of mechanical energy can be used to work out unknown velocities or heights, as shown in the examples below.

There are a number of slightly different approaches to doing conservation of energy calculations; here we will always use:
Total mech. energy at the beginning = Total mech. energy at the end.

Example 2. A ball of mass 50g is thrown vertically upwards from ground level with speed 30ms^{-1}.
- ***a) Find the maximum height to which it rises, taking g = 9.81ms^{-2}***
- ***b) Find its speed when it is 10m above ground level***
- ***b)Give one assumption that you have used in your calculation.***

a)We will take potential energy as 0 at ground level

> ***Tip**. It is helpful to make a definite decision where to take potential energy as zero, since it is then easier to make sure you always measure from the same place.*

Total mechanical energy at start $= g.p.e. + k.e.$
$= mgh + \frac{1}{2}mv^2$
$= 0 + \frac{1}{2} \times 0.05 \times 30^2$

> ***Tip**. To avoid rounding or copying errors, do not work out the actual figures until the end*

Total mechanical energy at end $= 0.05 \times g \times h + 0$

> ***Tip**: Remember the ball will rise until its velocity is zero. You often need to use this idea in this sort of problem.*

Conservation of mechanical energy gives:

$0 + \frac{1}{2} \times 0.05 \times 30^2 = 0.05 \times g \times h + 0$

So $h = \dfrac{\frac{1}{2} \times 0.05 \times 30^2}{0.05 \times 9.81} = 45.87....m = 46m$ (2 SF)

b) Total energy at start = $\frac{1}{2} \times 0.05 \times 30^2$, *as before*
Total energy when 10m above ground = $0.05 \times g \times 10 + \frac{1}{2} \times 0.05 \times v^2$
So $\frac{1}{2} \times 0.05 \times 30^2 = 0.05 \times g \times 10 + \frac{1}{2} \times 0.05 \times v^2$

So $v^2 = \dfrac{\frac{1}{2} \times 0.05 \times 30^2 - 0.05 \times 9.81 \times 10}{\frac{1}{2} \times 0.05} = 703.8$

$\Rightarrow v = \sqrt{703.8} = 27\ ms^{-1}$ (2SF)

c) Since we have used conservation of mechanical energy, we are assuming that no external forces act. So in this case, we are assuming there is no air resistance.

Example 3. A book is dropped from a height of 4 metres. Taking g = 10ms^{-2}, find its speed when it hits the floor

a) We will take potential energy as 0 at floor level.

Total mechanical energy at start $= mg \times 4 = 4mg$
Total mechanical energy at end $= \frac{1}{2}mv^2$

You might be tempted to panic because we don't have a value for m. But it will cancel out, since it occurs on both sides of the equation:

Using conservation of mechanical energy:

$4mg = \frac{1}{2}mv^2$

So $8g = v^2$

$v = \sqrt{80} = 8.9\ ms^{-1}$

4. The Work-Energy Principle

When external forces are involved, conservation of mechanical energy cannot be used. Instead we use the **work-energy principle**.

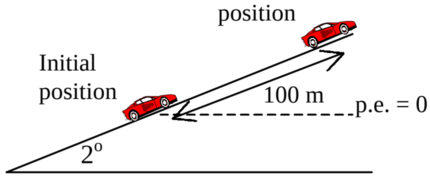
> *Work done by external force = increase in total mechanical energy + work done against friction*

The following examples show how to use this:

Example 4.
A car of mass 800kg is at rest on a road inclined at 2^o to the horizontal. Later, it is 100m further up the road, travelling at 10 ms^{-1}. Assuming there are no frictional resistances to motion, find the average driving force exerted by the car's engine. Take g = 9.8ms^{-2}

Note: we know we cannot use conservation of mechanical energy because the car has gained both kinetic and gravitational potential energy.

Step 1. Draw a diagram, & decide where to take potential energy as zero



Step 2. Work out initial & final total energy

Total initial energy = 0

Final k.e. $= \frac{1}{2} \times 800 \times 10^2$

To find the final g.p.e., we need to work out the vertical height of the car above its initial position

Using trigonometry, we find $h = 100\sin 2^o$
So final g.p.e = $800 \times 9.8 \times 100\sin 2^o$

So total final energy = 67361J

Step 3. Use the work energy principle, putting in everything we know

WD by external force = inc in mechanical energy + WD against friction

There is no work done against friction, since we are told there is no frictional force.

The only external force involved is the driving force, D, which acts directly up the hill, in the same direction as the distance moved.

So we have: $D \times 100 = 67361 - 0$
So $D = 673.61N =$ ***670N (2SF)***

> ***Exam Hints:***
> *1 Ensure you write down the equation you are using - this will probably gain you a "method" mark, even if you make a careless mistake.*
>
> *2 Do not lose out by using too many or too few significant figures - you should never use more SF than the question does*

Typical Exam Question

(a)(i) Write down an equation for W, the work done when force F moves an object a distance s, in a direction that makes an angle θ° to the line of action of the force. [1]

(ii) Use this equation to derive an expression for the gain in gravitational potential energy when a mass m is raised through a height h. [2]

(b)(i) Describe the energy changes taking place when a person jumps upwards from standing. [2]

(ii) If the person has achieved a speed of 6.1ms^{-1}, calculate the maximum height that they can jump. [2]

(iii) What assumption have you made in this calculation? [1]

(iv) If the take off speed could be increased by 10%, would there be a 10% increase in the height jumped? Explain. [2]

(a)(i) $W = Fs\cos\theta$ ✔

(ii) Gain in GPE = work done in raising mass through height h ✔

$F = mg \quad s = h$ *and* $\theta = 0$ *as force and distance are vertical*

$W = mgh$ ✔

(b)(i) Chemical energy from food is changed into kinetic energy and heat in her muscles ✔

Her kinetic energy is then transformed into gravitational potential energy. ✔

(ii) Kinetic energy is transformed into gravitational potential energy:

$\frac{1}{2}mv^2 = mgh$ ✔

$h = v^2/2g = 6.1^2/2 \times 9.8 = 1.9m$ ✔

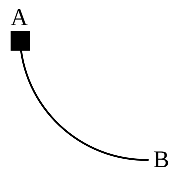
(iii) The calculation assumes that all of the kinetic energy at take off is transformed into gravitational potential energy and that there are no other energy transformations. ✔

(iv) No, since KE = $\frac{1}{2}mv^2$ *,a 10% increase in speed would produce an energy increase of more than 10%.* ✔

$mgh = \frac{1}{2}mv^2$ *so the height is proportional to the KE*

The increase in height will also be more than 10% ✔

Example 5. The diagram below shows a curved track in the form of a quarter of a circle of radius 10cm.
A small block of mass 30 grammes is held at point A, then released. Given that the average frictional force between the block and the track is 0.1N, find the speed of the block when it reaches B. Take g = 10ms^{-2}



Taking potential energy as 0 at B:

Initial g.p.e. = $0.03 \times 10 \times 0.1 = 0.03J$ *Initial k.e. = 0.*
So initial mechanical energy = 0.03J
Final g.p.e. = 0 *Final k.e. =* $\frac{1}{2} \times 0.03 \times v^2$
So final mechanical energy = $0.015v^2$

Work done against friction = frictional force × distance moved
Distance moved = quarter of circle = $\pi(0.1)^2 \div 4 = 0.0025\pi$.
So work done against friction = $0.1 \times 0.0025\pi = 0.00025\pi$.

Now use the equation. There are no external forces acting (except friction)

$0 = 0.015v^2 - 0.03 + 0.00025\pi$

So $v^2 = \dfrac{0.03 - 0.00025\pi}{0.015} = 1.9476....$ *So* $v = 1.40ms^{-1}$ *(3SF)*

Tip *Increase in mechanical energy is* ***always***
final energy – initial energy.

Typical Exam Question

A toy car of mass 0.3kg is moving along a horizontal surface. It is travelling at 4ms^{-1} when it reaches the foot of a ramp inclined at an angle of 20° to the horizontal.

(a) (i) Calculate the kinetic energy of the car at the instant it reaches the foot of the ramp [1]

(ii) The car rolls up the ramp until it stops.
Ignoring resistive forces, calculate the vertical height through which the car will have risen when it stops. Take *g= 9.8ms^{-2}*[2]

(b) In practice the car only rises by 75% of this theoretical value. Calculate

(i) the energy lost in overcoming resistive forces. [1]

(ii) the average resistive force acting on the car, parallel to the slope of the ramp. [4]

(a) (i) $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.3 \times 4^2 = 2.4J$ ✔

(ii) $mgh = 2.4J$ ✔

$h = 2.4 / (0.3 \times 9.8) = 0.82m$ ✔

(b) (i) Energy lost in overcoming friction = 25% of 2.4J = 0.6J ✔

(ii) Frictional force × distance moved=WD against friction=0.6J ✔

Distance moved up slope = s

Where, $s\sin 20° = (75/100) \times 0.82$ ✔ $s = 1.8m$ ✔

Frictional force × $1.8 = 0.6$ *so friction=0.6 / 1.8=0.33N* ✔

5. Power

Power is the rate of doing work.
If work is being done at a steady rate, then:

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

The unit of power is the ***watt****; 1 joule of work being done per second gives a power of 1 watt.*

Example 6. A pump raises water through a height of 4.0m and delivers it with a speed of 6.0ms^{-1}. The water is initially at rest.
The pump moves 600kg of water every minute. Taking g = 9.8ms^{-2}, calculate the power output of the pump. (You may assume that the pump works at a steady rate)

First, we need to find the work done.

Every minute, the pump raises 600kg of water through 4.0m, and gives it a speed of 6.0 ms^{-1}
So every minute:

increase in kinetic energy = $\frac{1}{2} \times 600 \times 6^2 = 10800J$

increase in gravitational potential energy = $600 \times 9.8 \times 4 = 23520J$

So increase in mechanical energy per minute = 10800 + 23520 = 34320J

So the pump must do 34320J of work on the water every minute.
So power of pump = 34320/60 = 572W = 570W (2SF)

Tip*. Be careful to always use SI units – in the above example, it might have been tempting to work in minutes, but seconds are what is required by the definition of the watt.*

There is also another form of the power formula which is particularly useful when working out the power of a machine - like a car or a train – exerting a **constant driving force**.

We know: work done = Force × distance moved in the direction of force.
So, if the driving force is D, we have work done = Ds
If work is done at a constant rate, and the velocity is constant we have:

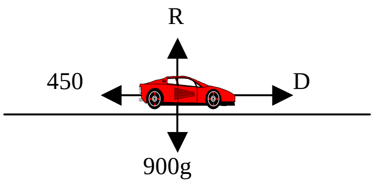
$$\text{Power} = \frac{Ds}{t} = D \times \frac{s}{t} = Dv$$

Power = Driving Force × velocity

When using the equation P = Dv
When using the equation P = Dv, you often also need to use
- resolving, to find the driving force
- the idea that if something is moving with constant velocity, the resultant force on it is zero.

Example 7. A car of mass 900kg can travel at a maximum speed of 20 ms^{-1} on a level road. The frictional resistance to motion of the car is 450N.
a) Find the power of the car's engine.

The car is travelling up a road inclined at sin^{-1}(0.02) to the horizontal with its engine working at the same rate. Find its maximum speed. Take g = 9.8ms^{-2}



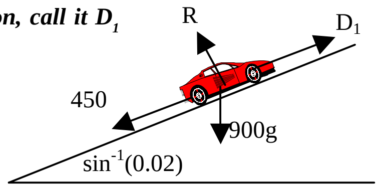
a) Step 1. Draw a diagram, showing all forces.
Step 2. Resolve in the direction of motion.
Note: if the car is at maximum speed, there is no resultant force

$D - 450 = 0 \Rightarrow D = 450$

Step 3. Use the power equation
$P = Dv = 450 \times 20 = 9000W$

b) Note that "engine working at the same rate" means "exerting the same power" ***It does NOT mean that D is the same. To avoid confusion, call it D_1***

1.



2. We must resolve up the slope, since that's the direction the car is moving in.
$D_1 - 450 - 900g\,(0.02) = 0$
$D_1 = 626.4N$

3. $P = Dv \Rightarrow 9000 = 626.4v \Rightarrow v = 14\ ms^{-1}$ *(2 SF)*

Typical Exam Question
(a) Show that the unit of power is equivalent to that of force × velocity. [2]
(b) On a straight, level road a cyclist with a power ortput of 95 W can cycle at a maximum steady speed of 5ms^{-1}. The combined mass of the cyclist and cycle is 90kg. Calculate:
(i) The total resistive force exerted on the cyclist. [2]
(ii) Assuming that the resistive force remains the same, calculate the maximum speed the cyclist can maintain up a slope of 1 in 50. [4]

(a) Units of force × velocity = N × (m / s) ✓
Units of power = J / s = N m / s ✓
(b) (i) Force = power / speed ✓*= 95 / 5 = 19N* ✓
(ii) Assume V ms^{-1} is the maximum speed.
Work done in one second against resistive force = power = force × velocity = 19 × V = 19V joules ✓
Work done in 1 second raising the gpe of the cyclist = mgh = 90 × 9.8 × V / 50 = 17.6V joules ✓
Total work done per second = 19V + 17.6V = 36.6V joules
36.6V = 95 ✓ *so V = 95 / 36.6 = 2.6ms^{-1}* ✓

Efficiency

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

No real machine is 100% efficient – there is always some "lost" energy.

Of course, this "lost" energy is actually converted to another form – usually heat, generated by friction. But it is not possible to make use of it.

Questions involving efficiency may require you to:
- work out the efficiency of a machine
- work out the energy input, when you know the efficiency and the energy output
- work out the energy output, when you know the efficiency and the energy input.

Typical Exam Question
(a) When an athlete is performing press-ups, the average force in each arm is 200N. Calculate the work done by his arms during one press-up, which raises his shoulders 0.50m above the ground. [2]
(b) If the athlete can do 16 press-ups per minute, calculate the:
(i) total power output of his arms. [2]
(ii) energy input to his arms in one minute, if the overall efficiency of his arms is 20%. [2]

(a) Work = force × distance = 200 × 0.5 ✓*= 100J per arm*
Total work = 200N ✓
(b) (i) Power output = work done / time taken = 16×200 / 60 ✓
= 53.3W ✓
(ii)(power output / power input) × 100 = 20
pwr input = pwr output × 100 /20=53.3×100 / 20 ✓*=266.5W*
Energy input in one minute = 266.5 × 60 = 1.6 × 10^{4}J ✓

Questions
1. Define the *joule* and the *watt*.
2. Explain what is meant by *efficiency*.
3. Explain what is meant by the *principle of conservation of mechanical energy*, and give the circumstances in which it can be used.
4. Is work a scalar or a vector?
5. A force of magnitude 6N moves a body through a distance of 2 metres. Find the work done by the force if
   a) the distance moved is in the direction of the force
   b) the distance moved is inclined at 60° to the direction of the force.
6. A ball is thrown vertically upwards from ground level with speed 20ms^{-1}. Taking g = 10ms^{-2},
   a) Find the height to which it rises.
   b) Find its speed when it is 2m above ground level.
   In the above calculation, you have neglected a force
   c) What is the name of this force?
   d) Would you expect your answer to a) to be larger or smaller if this force were taken into account? Explain your answer.
7. A marble is dropped from a height of 80cm above the ground.
   a) Neglecting air resistance, and taking g = 10ms^{-2}, find its speed as it reaches the ground.
   The ground that the marble falls on is muddy, and the marble sinks 2cm into the mud.
   b) Given that the mass of the marble is 10g, find the average force exerted by the mud on the marble.
8. A car of mass 800kg has a maximum speed of 20ms^{-1} down a slope inclined at 2° to the horizontal. The frictional resistance to motion of the car is constant, and of magnitude 1000N. Take g = 10ms^{-2}.
   a) Find the power of the car's engine
   b) Find its maximum speed up the same hill.

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

(a) Define:

(i) power. **[1]**

P = W/t

0/1

> Using an equation is fine, but only provided you define the symbols used in it.

(ii) the watt. **[1]**

1 watt = 1 joule per second ✓

1/1

> Although this was awarded the mark, it might have been better to say "1 joule of work is done per second" or "1 joule of energy is transferred per second"

(b) A car of mass 1200kg accelerates from rest along a straight, level road. If the car has a constant acceleration of 2.50ms^{-2}, calculate:

(i) the force causing this acceleration. **[1]**

3000N✓

1/1

> Although the candidate's answer is correct, s/he would be wise to show their working and include the formula used, to ensure full credit is gained.

(ii) the work done in moving the car the first 100m. **[1]**

300000

0/1

> Although the answer is numerically correct, the mark was not awarded because the candidate omitted the units.

(iii) the average power output of the car during the first 100m **[3]**

s = ½ at^2.

100 = 1.25t^2. t = √80 = 8.944271 ✓

Power = 300000/8.944271 ✓= 33541.02 W

2/3

> Candidate was not awarded the final mark, since the number of significant figures given in the answer was far too great.

Examiner's Answers

(a) (i) Power = work done / time taken ✓

or: power is the number of joules of energy converted from one form to another in one second

(ii) Power of 1 watt is developed when 1 joule of work is done in 1 second ✓

(b) (i) F = ma = 1200 × 2.50 = 3000N ✓

(ii) Work done = force × distance = 3000 × 100 = 3 × 10^{5}J ✓

(iii) Power = Work Done / time taken ✓

s = ut + ½ at^2 u =0

So: s = ½ at^2

Giving time taken as: t = √(2s/a)

= √(2 × 100 / 2.5) = 8.94s ✓

Power = 3 × 10^5 / 8.94 = 3.36 × 10^{4}W ✓

Answers to questions

1. – 4. can be found in the text

5. a) 12J b) 6J

6. a) 20m. b) 19.0ms^{-1} c) air resistance

d) smaller, because the total mechanical energy will have decreased.

7. a) 4ms^{-1} b) 4N

8. a) 14400W b) 11.3ms^{-1}

Acknowledgements;

This Physics Factsheet was researched and written by ***Cath Brown***

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Physics Factsheet

www.curriculum-press.co.uk Number 135

Energy Changes for Vertical Motion

- What energy changes occur when an object falls?
- How is the original height of an object related to the speed before impact?
- What is the effect of air resistance on these energy changes?

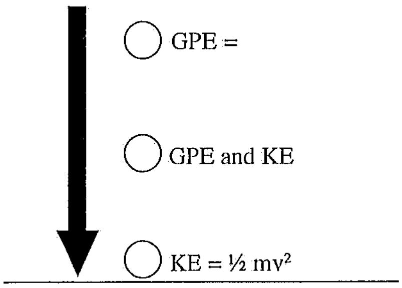
A moving object has kinetic energy, $KE = 1/2mv^2$. When an object changes height, there is a change in gravitational potential energy, $\Delta GPE = mg\Delta h$.

Relate these two expressions for a falling object $1/2mv^2 = mg\Delta h$. Then rearrange for speed.

$1/2v^2 = g\Delta h$ $\quad v^2 = 2g\Delta h$ $\quad v = \sqrt{(2g\Delta h)}$

So, with negligible air resistance, the speed of a falling object is **only** dependent on the change in height, and NOT the mass of the object.

Vertical motion energy changes



The speed of a falling object does NOT depend on the mass of the object (negligible air resistance)

Exam Hint: *Try to rearrange the KE and GPE equations yourself to produce $v = \sqrt{(2g\Delta h)}$*

Worked example 1

(a) Rearrange the kinetic energy and gravitational potential energy equations to show the relationship between the speed and change in height for a falling object assuming negligible air resistance (4 marks).

(b) (i) Determine the speed of a 1kg mass that falls 13m (2 marks)

(ii) State the speed of a 1g mass that also falls 13m (1 mark).

(c) State and explain the energy changes for a falling object that hits the ground (2 marks)

Answers

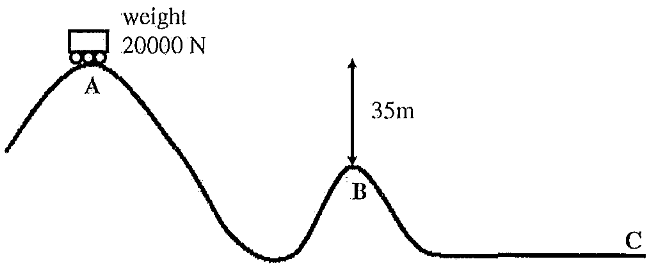
(a) See above for this mathematical proof.

(b)(i) $v = \sqrt{(2g?h)} = \sqrt{(2 \times 9.81Nkg^{-1} \times 13m)} = 16.0ms^{-1}$

(ii) $16.0ms^{-1}$

(c) Before the object is dropped, it has only GPE. This becomes KE as it falls. When the object hits the ground and comes to rest, this KE becomes thermal energy, warming the surroundings.

Worked example 2



A roller coaster moves from A to C.

(i) Determine the change in GPE of the roller coaster from A to B.

(ii) Calculate the maximum speed of the roller coaster at B.

(iii) Explain why the speed of the roller coaster is likely to be less than this maximum.

Answers

(i) $\Delta GPE = mg\Delta h$ Change in gravitational potential energy $= 20{,}000N \times 35m = 7.0\times10^5J$.

(ii) $v = \sqrt{(2g?h)} = \sqrt{(2 \times 9.81N/kg \times 35m)} = 26.2ms^{-1}$

(iii) The maximum speed assumes that there is a 100% transfer of GPE to KE. In reality, some energy would be transferred into other forms, such as heat due to the force of friction between rails and wheels.

Exam Hint: *Take care when dealing with mass or weight. $\Delta GPE = mg\Delta h$ OR ΔGPE = weight $\times \Delta h$*

Just before an object hits the ground, all of the gravitational potential energy has become kinetic energy. When it hits, this kinetic energy becomes heat energy, the object and surroundings become warmer.

When a falling object comes to rest, all of the original GPE becomes thermal energy, warming the surroundings.

In reality, as objects fall faster, air resistance has a bigger effect. When the air resistance balances the weight of an object, it travels at a steady speed. How does this affect the energy changes? The object still loses gravitational potential energy as it falls. However, it is NOT gaining kinetic energy, as it is not falling more quickly.

Where does this GPE go?

The object is applying a force on the air particles. This causes heating, warming the air and object up as it passes.

If a falling object travels at a steady speed, gravitational potential energy is converted into heat energy.

Exam Hint: *When an object falls at a steady speed due to air resistance, the loss in GPE is EQUAL to the work done on the air particles.*

Worked example 3

(a) A 1kg mass falls 15m at a constant velocity.
(i) Calculate the change in gravitational potential energy for the mass (2 marks).

(ii) State and explain how much work is done on the air particles by the falling mass (3 marks).

(b) Explain why the object is travelling with constant velocity, quoting an appropriate Newton's Law of motion (2 marks).

Answer

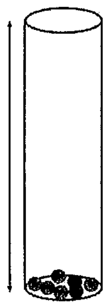
(a) (i) $\Delta GPE = mg\Delta h$ *Change in gravitational potential energy* $= 1kg \times 9.81N/kg \times 15m = 147.2\ J$

(ii) 147.2 J. The mass is not accelerating so there is no gain in kinetic energy. The gravitational potential energy is transferred into thermal energy in the air. The falling mass does work against the air particles.

(b) The upward force of air resistance balances the downward force of the weight; Newton's first law of motion states that an object will travel at a constant velocity (which could be zero) unless an unbalanced force acts.

Worked example 4

An experiment is carried out with 50g of lead in a 1m sealed tube. The tube is inverted 100 times.



Calculate the temperature change for the lead. The specific heat capacity of lead is $130Jkg^{-1}K^{-1}$***. Assume no energy transfer to the surroundings.***

Answer

The lead effectively falls 100m.
The corresponding loss in GPE = mgh
$= 0.05kg \times 9.81Nkg^{-1} \times 100m = 49.05J.$
This warms up the lead.

The relationship for specific heat capacity;

Energy change = mass × temperature change × specific heat capacity.

$49J / (0.05kg \times 130Jkg^{-1}K^{-1}) = 7.5K.$

Practice Questions

1. (a) Calculate the change in gravitational potential energy for 1kg of water in a 9m waterfall. (1mark)

   (b) Calculate the temperature change of water due to this waterfall. The specific heat capacity of water is $4200Jkg^{-1}K^{-1}$. Assume no energy is transferred to the surroundings. (2 marks)

2. (a) Determine the velocity of a rollercoaster due to a 45m drop. (Assume zero velocity at the top of the drop.) (2 marks)

   (b) Explain why the velocity is likely to be different in reality. (2 marks)

3. (a) A 1.5m tube containing 12g of lead shot is inverted 25 times. Calculate the corresponding temperature change of the lead. The specific heat capacity of lead is $130Jkg^{-1}K^{-1}$ (3 marks)

   (b) State what assumptions you have made in this calculation.

Acknowledgements:
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Answers
1) a) 88.3J b) 2.1×10^{-2} K
2) a) $29.7ms^{-1}$
3) a) 0.11K

Physics Factsheet

www.curriculumpress.co.uk Number 61

Answering questions using conservation of mechanical energy

The purpose of this Factsheet is to give guidance on how to approach AS and A2 questions involving conservation of mechanical energy.

Before studying the Factsheet, you should make sure that you are familiar with the concept of conservation of energy and all the important applications of mechanical energy such as: mass/spring system (Factsheet 53); simple pendulum (Factsheet 54), collisions and projectiles. You should also be familiar with the expressions for kinetic energy (K.E.) and gravitational potential energy (G.P.E.)

- You should know that collisions in which K.E. is conserved are called **elastic** collisions.
- It is important to appreciate that you are unlikely to get a question which concerns **only** conservation of mechanical energy. It is highly likely that forces, distances and speeds will also be involved.
- It is not always obvious that you need to use conservation of mechanical energy in order to answer a question. In questions asking about speed, it is often used through the relationship K.E. = ½ mv^2, so if a known amount of G.P.E has been changed into K.E (assuming no other transfer) then the speed of the object can be calculated.

Example 1. A ball of mass 150g is dropped from a height of 3m onto the ground. Calculate the speed of the ball when it hits the ground. Candidates often think that this question requires the use of equations of motion, but it is better approached from the viewpoint of conservation of energy.

Assuming no external energy transfers, G.P.E. lost = K.E gained
G.P.E lost = 0.15 × 9.81 × 3 = 4.4145J, this energy gives ½mv^2

So $\frac{1}{2} \times 0.15 \times v^2 = 4.4145$

$$v^2 = \frac{4.4145 \times 2}{0.15} = 58.86$$

So v = 7.67m/s

In fact the mass was unnecessary, since this calculation could have been done in one step with the mass cancelling out.
$\frac{1}{2} mv^2 = mgh \qquad v^2 = 2 \times g \times h$

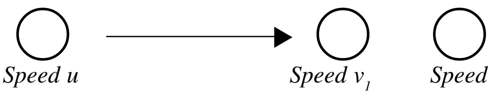
Example 2. Calculate the speed which a ball must be thrown vertically if it is to reach a height of 4m.

Assuming all the K.E is transferred to G.P.E, then
K.E. lost = G.P.E. gained
since at maximum height its speed is zero.

$$\text{So} \quad \tfrac{1}{2} mv^2 = mgh$$
$$v^2 = 2 \times 9.81 \times 4$$
$$v = 8.85 \text{ m/s}$$

- It is also highly likely that you will need to use the principle of conservation of momentum in the same question, since the use of conservation of momentum and conservation of K.E. in elastic collisions is a powerful tool in solving mechanics problems. (You should remember that momentum = $m \times v$)

Example 3 A ^{4}He nucleus travelling with speed u has a head-on collision with a particle of the same mass, which is at rest. If the collision is elastic, show that the incident particle comes to rest and the target particle moves off with the same speed that the incident particle had i.e. that the incident particle transfers all its K.E. to the target particle



Conservation of momentum gives:
$$mu = mv_1 + mv_2 \qquad (a)$$

Conservation of K.E. gives:
$$\tfrac{1}{2} mu^2 = \tfrac{1}{2} mv_1^2 + \tfrac{1}{2} mv_2^2 \qquad (b)$$

(a) gives $u = v_1 + v_2$ *(the m's cancel)*

Substituting for u in (b) with the ½ m's cancelled gives:
$$(v_1 + v_2)^2 = v_1^2 + v_2^2$$
$$v_1^2 + 2v_1v_2 + v_2^2 = v_1^2 + v_2^2$$

The only way that this equation can be satisfied is if $2v_1v_2$ *is zero, so either* v_1 *or* v_2 *must be zero.* v_2 *cannot be zero if* v_1 *is non-zero, because the incident particle would have to travel through the target particle, so the only realistic solution is that* v_1 *is zero. Thus* v_2 *must equal* v_i *i.e. the incident particle has transferred all its energy to the target particle.*

Note that this result is independent of the initial energy or mass.

Note also that momentum is a vector quantity, but here the velocities are all along the same straight-line, so vector addition is not necessary. In fact if the velocities are not all in the same straight line, the vector treatment of the equations above leads to the fact that the only way they can be satisfied is if the two particles move off at right angles to each other. This is observed in collisions between alpha particles in a cloud chamber filled with helium gas.

Example 4. A toy railway truck of mass 1kg, moves at a speed of 2ms^{-1} towards a second stationary truck of mass 2kg along a smooth track.
(a) If the incident truck moves backwards with a speed of $^2/_3$ ms^{-1} after the collision, calculate the speed of the target truck after the collision.
(b) Show that the collision is elastic.

(a) Let the target truck move off with speed v.
Conservation of momentum gives:
$1 \times 2 = -1 \times {}^2/_3 + 2v$ *(Remember velocity is a vector quantity, so moving backwards is written as minus)*
$$2v = 2 + {}^2/_3 = {}^8/_3$$
$$v = {}^4/_3$$
The target truck moves forwards with speed $^4/_3$ ms^{-1}

(b) K.E. before the collision $= \tfrac{1}{2} \times 1 \times 4 = 2J$
K.E. after the collision $= \tfrac{1}{2} \times 1 \times {}^4/_9 + \tfrac{1}{2} \times 2 \times {}^{16}/_9$
$= 2J$
Since K.E. is conserved, the collision is elastic.

- You may also be asked questions in which the assumption that mechanical energy is conserved does not quite hold, and you may be asked to suggest why not.

Example 5: Passengers on a Ferris Wheel travel in a circle of radius 60m at a steady speed of about 12ms^{-1}.
(a) what is the change in the passengers' velocity between the bottom and the top of the wheel?
(b) A particular passenger has a change of G.P.E of 60KJ between bottom and top. What is her mass?
(c) The motor driving the wheel does not have to supply this energy, provided the ride is full. Explain why not.
(d) The motor does have to supply energy, though. Explain why.

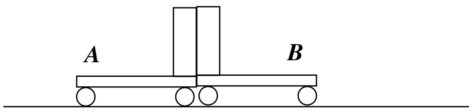
The first part of the question is to check your recollection that velocity is a vector quantity.
(a) If the Ferris wheel is moving anticlockwise, and taking left-to-right as the positive direction, then the velocity at the bottom is + 12ms^{-1} and that at the top is –12ms^{-1} so the change in velocity is 24ms^{-1}
(b) Change in G.P.E. = $mg\Delta h$, so $6 \times 10^4 = m \times 9.81 \times 120$

$$m = \frac{6 \times 10^4}{9.81 \times 120} = 60.0\text{kg (3.s.f)}$$

(c) Provided the ride is full, then for every passenger who gains G.P.E. another loses the same amount, so no energy needs to be supplied.
(d) Not all the energy is exchanged between G.P.E. and K.E., some is transformed into thermal energy in doing work against friction between the moving parts, some is transformed into thermal energy in doing work against air resistance and some does work in moving the structure itself.

- Questions often pose situations in which mechanical energy is **not** conserved, so it is important to recognize these as well.

The diagram shows two trolleys, initially at rest, in contact with each other on a smooth horizontal bench.
A spring-loaded piston is released which pushes the trolleys apart.



(a) State the total momentum of the trolleys as they move apart. Explain your answer.
(b) A has a mass of 0.8kg, B a mass of 1.2kg. If B moves off with a velocity of 1.5ms^{-1}, calculate the velocity of A.
(c) Calculate the total K.E. of the trolleys. Since the initial K.E. was zero, K.E. has not been conserved. Where has this K.E. come from?

(a) The total momentum of the trolleys as they move apart is zero. The initial momentum was zero. Momentum is conserved whenever there are no external forces acting. Being told that the bench is smooth entitles you to assume that no external forces are acting, though in practise this is not quite true, since there is still air resistance. Since momentum is a vector quantity, one trolley now has + momentum and one –, thus this can add up to zero.

(b) $M_BV_B + M_AV_A = 0$
$1.2 \times 1.5 + 0.8 \times V_A = 0, \quad 0.8 \times V_A = -1.8$
$V_A = -2.25\text{ms}^{-1}$
The minus sign is important, since it indicates that trolley A moves backwards.

(c) $K.E_A = \frac{1}{2} M_AV_A^{\ 2} = 0.5 \times 0.8 \times 2.25^2 = 2.025\text{J}$
$K.E_B = \frac{1}{2} M_BV_B^{\ 2} = 0.5 \times 1.2 \times 1.5^2 = \underline{1.35\text{J}}$
Total K.E. = $\underline{3.375\text{J}}$
This K.E. has been transformed from elastic energy which was stored in the spring

Exam Workshop
This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's mark scheme is given below.

A boy, of mass 45kg, is sliding on ice at a constant 3ms^{-1}. When he collides with another boy of mass 50kg who is travelling at 2ms^{-1}, they become entangled and move off together.
(a) Calculate the speed with which they move off, stating any assumption you make. (3)
Momentum is conserved.
$M_1V_1 + M_2V_2 = M_tV_t$
$(45 \times 3) + (50 \times 2) = 95 \times V_t$
$135 + 100 = 95 \times V_t$
$V_t = 2.47\text{ms}^{-1}$ *2/3*

The candidate has scored 2 marks for a correct calculation, but conservation of momentum is not an assumption. The assumption is that there are no external forces acting, in which case momentum is conserved.

(b) Discuss the extent to which the assumption is true. (2)
The principle of Conservation of Momentum states that momentum is always conserved. *0/2*

The candidate has missed the point of the question, which was to test understanding of the conditions in which the Principle of Conservation of Momentum can be used.

(c) This collision is not elastic, calculate the change in K.E.(3)
K.E before = $(\frac{1}{2} \times 45 \times 9) + (\frac{1}{2} \times 50 \times 4) = 302.5\text{J}$
K.E after = $\frac{1}{2} \times 55 \times 2.47^2 = 167.8\text{J}$
Change in energy = 134.7 J
The K.E. has decreased by 134.7J *3/3*

The candidate has performed the calculation correctly.

(d) What has happened to the apparently "lost" energy? (2)
It has been dissipated as thermal energy. *0/2*

A lot of candidates' first, unthinking, response to energy "loss" is to quote energy dissipated as thermal energy. Here the assumption has been made in calculating the final speed that momentum is conserved. That requires no external forces acting, hence no friction, so no energy is dissipated as thermal energy.

Examiner's Answers
(a) assumption is that there are no external forces acting, in which case momentum is conserved.
$M_1V_1 + M_2V_2 = M_tV_t$
$(45 \times 3) + (50 \times 2) = 95 \times V_t$
$135 + 100 = 95 \times V_t \qquad V_t = 2.47\text{ms}^{-1}$
(b) The assumption that there are no external forces acting is a reasonable approximation on ice, where the frictional force is very low.
(c) K.E before = $(\frac{1}{2} \times 45 \times 9) + (\frac{1}{2} \times 50 \times 4) = 302.5\text{J}$
K.E after = $\frac{1}{2} \times 55 \times 2.47^2 = 167.8\text{J}$
Change in energy = 134.7 J
The K.E. has decreased by 134.7J
(d) The internal energy of the boys has been altered in the collision.

Acknowledgements:** This Physics Factsheet was researched and written by **Janice Jones.
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Physics Factsheet

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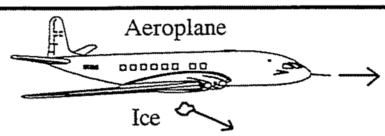
Difficulties with Motion and Energy problems

Examiners' reports point out repeated difficulties students experience with problems involving motion and energy. We will look at a sample question of this sort, and discuss the techniques that should be used.

Here is the problem we will consider:

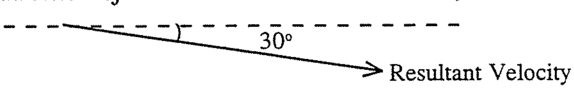


(a) A chunk of ice of mass 3.0kg falling from an aircraft's wing soon reaches terminal velocity in the vertical component of its motion. Explain why its vertical velocity becomes constant. (3 marks)

(b) The constant vertical velocity reached is 22.5ms^{-1}. Find the kinetic energy due to the vertical motion of the ice block. (3 marks)

(c) Gravitational force is exerted on the ice block as it falls through a distance. From W = Fs, we know that work is being done. Describe and explain the energy transfer that occurs. (2 marks)

(d) When the chunk of ice reaches its vertical terminal velocity, its direction of motion is 30° below the horizontal.



Calculate its horizontal velocity, v_x, at this time.

Part (a)

A chunk of ice of mass 3.0kg falling from an aircraft's wing soon reaches terminal velocity in the vertical component of its motion. Explain why its vertical velocity becomes constant. (3 marks)

Mark Scheme

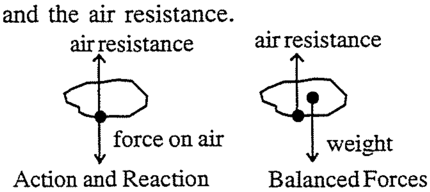
Any 3 marks from-
gravity/weight causes a downward acceleration (1)
air resistance/drag/friction acts upwards (1)
the upward force increases with the speed of the block (1)
zero acceleration/uniform velocity reached when forces balance (1)

A candidate's response

Gravity acts on the ice and it accelerates downwards. When it gets fast enough, its acceleration is cancelled by the air resistance, and it pushes as hard on the air as the air does on the ice. Constant velocity is reached.

Comments

A confused answer. A **force** must be balanced by **another force**, not by motion such as acceleration. Newton's Third Law (action and reaction) is always obeyed, but is not relevant to this question. The forces that must balance are the two forces acting **on the ice** – its weight and the air resistance.



In addition, the student was not precise enough as he didn't state the direction of the resistive force.

Marks gained: 1 / 3 (the first marking point only)

> *Action and reaction forces are always equal to each other. Two forces on a body will be equal in an equilibrium situation. You must be clear which situation you are dealing with.*

Part (b)

The constant vertical velocity reached is 22.5ms^{-1}. Find the kinetic energy due to the vertical motion of the ice block. (3 marks)

Mark Scheme

KE = ½mv² (1) = ½ x 3.0 x 22.5² (1) = 759J (1)

A candidate's response

KE = 0.5mv² = 0.5 x 3 x 22.5² = 33.75² = 1139W

Comments

The candidate has squared ½mv, rather than just the velocity. He has incorrectly stated the answer to 4 significant figures. He has also used the wrong unit for energy. This is just carelessness. Fortunately all of these errors occur in the same marking point – sometimes you get lucky.

Marks gained: 2 / 3

Part (c)

Gravitational force is exerted on the ice block as it falls through a vertical distance. From W = Fs, we know that work is being done. Describe and explain the energy transfer that occurs. (2 marks)

Mark Scheme

Any 2 marks from-
work done as a force has been exerted through a distance (1)
no gain in KE of block as velocity uniform (1)
kinetic/thermal energy gained by air as ice moves through it (1)

A candidate's response

As an object falls, gravitational potential energy is transformed into kinetic energy. Potential energy lost equals kinetic energy gained.

Comments

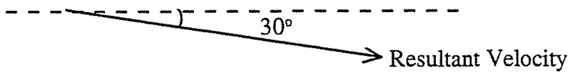
This candidate has not read the question. He has just stated the standard GCSE response where there is no air resistance. This question is quite clearly about work being done on an object where a resistive force is in play.
Beware of churning out stock answers to what appear to be standard situations. If you don't read the question, you're not likely to gain many, if any, marks.

Marks gained: 0 / 3

> *Exam Hint: Read the question carefully. Often standard situations are altered slightly to see if you can adjust your thinking.*

Part (d)

When the chunk of ice reaches its vertical terminal velocity, its direction of motion is 30° below the horizontal.



Calculate its horizontal velocity, v_x, at this time.

Mark Scheme

$tan\ 30° = \frac{22.5}{v_x}$ *(1)*

$v_x = \frac{22.5}{tan\ 30°}$ *(1)* $= 39.0ms^{-1}$ *(1)*

A candidate's response

tan è $= \frac{v_y}{v_x}$

$tan\ 30 = \frac{22.5}{v_x}$

$v_x = 22.5\ tan\ 30 = 13ms^{-1}$

Comments

The candidate has used the correct equation, substituted correctly, and used the correct unit in his answer. Unfortunately, carelessness in rearranging the equation has cost him dearly. It is easy to ignore this step when checking your calculation, but it is a common source of error.

Marks gained: 1 / 3

> ***Exam Hint:*** *When checking a calculation, it is as important to check that the numbers have been substituted correctly, and that the equation has been rearranged correctly, as it is to check the actual calculation done.*

Final comment

Questions of this sort involve forces, motion, and work/energy considerations. It is important to be clear which is most relevant in each part of the question. In this question, part (a) involves forces and motion, and part (c) involves work and energy. Analyse each part of the question before you attempt it.

An exercise for you to attempt

A candidate's answer to a question is given below. Identify three correct points that the candidate makes, and three incorrect or contradictory points.

Problem

A parachutist jumps from an aeroplane. She accelerates for a few seconds, then opens her parachute and decelerates quickly before continuing towards the earth at a uniform speed.

Explain her motion in terms of forces and energy. Consider only her vertical motion.

Candidate's answer

Her gravitational potential energy is greatest at the top. As she falls, this potential energy changes to kinetic energy, but her total energy is fixed. Friction with the air is transformed into heat energy.

The gravitational force acting on her is constant, regardless of her velocity or altitude. She accelerates because her weight is greater than air resistance. Opening her parachute does not affect her weight, but does affect air resistance and potential energy.

She slows down because air resistance is greater than weight, and finally falls with uniform speed as air resistance keeps increasing.

Solution

Correct points-
GPE greatest at the top
gravitational force (weight) constant
accelerates because weight greater than friction
parachute does not affect weight, but affects air resistance
slows as the air resistance is greater than the weight

Incorrect points-
PE changes to KE (only true at the beginning of the fall)
her total energy is fixed
friction transformed into heat energy (a force cannot become an energy)
opening the parachute affects the PE
air resistance keeps increasing at uniform speed

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Physics Factsheet

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Properties of Materials

The properties of materials are a way of describing what a material is capable of and how it will behave under certain conditions. This is important when choosing materials for certain jobs, for instance choosing building materials or finding a good electrical insulator for the handle of an electrician's screwdriver!

Properties and Definitions

Here is a quick reference list of the main definitions. They include definitions that would be acceptable if given in an exam. Where units are given they are SI units.

Mass	The amount of matter in a body measured in **kg** - or more properly, mass is a measure of an object's resistance to changing its velocity
Volume	The space taken up by a material in **m^3**
Density	The amount of mass per unit volume measured in **$kg\ m^{-3}$**
Young's modulus	A constant for a material given by stress/strain in **$N\ m^{-2}$** or **Pa**
(Tensile) Stress	The force per unit cross-sectional area of a material that is being stretched or compressed (hence the 'tensile') in **$N\ m^{-2}$** or **Pa**
(Tensile) Strain	The change in length/original length for a stretched/compressed material (which has no units as it is a ratio)
Elasticity	An elastic material will return to its original shape when stressing forces are removed.
Ductility	A ductile material is one that can be stretched but does not return to its original shape when the stress is removed
Hardness	How difficult it is to cut, dent or mark a material
Softness	A measure of how easily a material deforms without breaking
Brittleness	A brittle material will not stretch far beyond its elastic limit without breaking
Toughness	The amount of energy a material can absorb before it breaks.
Strength	How much force a material can withstand before it breaks
Malleability	How easily a material can be pressed, bent, hammered or rolled
Stiffness	The amount of resistance to being bent
Electrical conductivity	A material's ability to conduct electricity - dependent on the availability of free electrons
Thermal conductivity	A material's ability allow heat to flow through it
Sonorous	A sonorous material will make a ringing sound when struck
Lustrous	A lustrous material can be polished to a high shine
Melting point	The temperature at which a material changes from a solid to liquid, or a liquid to a solid (freezing)
Boiling point	The temperature at which a material will turn from a fluid to a gas, or gas to a fluid (condensing)
Latent heat	The energy needed to change the phase of a material
Heat capacity	The energy needed to raise the temperature of a material
Viscosity	A measure of how slowly a fluid will flow

Some of these larger topics are discussed in more detail on other factsheets while the main points of the rest are discussed here.

Density

The density of a material tells you how much mass you would find in each cubic metre. It depends upon the mass of the particles which make up the material and how closely they are arranged. For instance, most substances become more dense when they freeze as the arrangement of particles becomes more ordered and therefore closer together. Water is one exception; when it freezes its particles become more spaced out making it less dense.

The density of a material is given by: $\rho = \frac{m}{v}$

> Where ρ is the density in kg m^{-3}
> M is the mass in Kg
> V is the volume of the object in m^3

Most solids and liquids have a density of the order of magnitude of 1000 Kg m^{-3} while most gases have densities around 1000 times smaller than this.

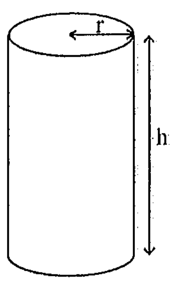
It helps to be able to work out volumes of various basic shapes although these will often be provided in the formula page in the exam:

e.g. the volume of a sphere is given by: Sphere = $^4/_3\pi r^3$
Where r is the radius

Prisms are shapes that have the same cross section all the way through. The volumes of such shapes are found by multiplying the area of the end face by the depth or height, for instance, the volume of a cylinder is given by: cylinder = $\pi r^2 h$

> Where h is the height of the cylinder
> and the ðr^2 is the area of the circular end face.

Fig 1a. volume of a cylinder with radius, r and height h.



> ***Example 1:***
> ***The density of copper at room temperature is about 9 000 Kg m^{-3}.***
> ***(i) Calculate the mass per metre of a copper wire that is 1mm in diameter.***
> ***(ii) What would be the mass of 2.5m of this wire?***
>
> **Answer**
> (i) Area of wire = $\pi r^2 = \pi(1\times10^{-3}/2)^2 = 7.85\times10^{-7}$ m^2
> Density per unit length = density × area
> $= 9000 \times 7.85\times10^{-7}$
> $= 7.065\times10^{-3}$
> $= 7.1\times10^{-3}$ kg m^{-1}
> (ii) Mass = mass per unit length × length
> $= 7.1\times10^{-3} \times 2.5$
> $= 17.8\times10^{-3}$ kg

Density in Liquid and Gases

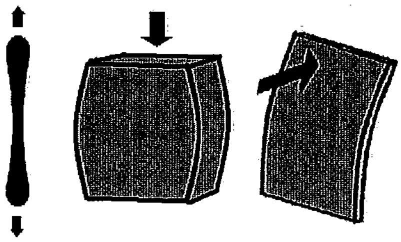
Liquids cannot be compressed and so occupy a fixed volume but as they can flow they do not maintain a fixed shape. This means that the density of a liquid does not change regardless of what shape it takes.

Gases expand to fill whatever container they are put in. As such they have no fixed shape and their volume depends upon the volume of the container. Therefore the density is also not fixed and depends upon the volume of the container.

Deformation

When a force is applied to a material, we say that a stress has been applied to it. This can result in a change in shape which we call deformation. How a material acts under stress depends upon its properties.

Fig 2. deforming materials results in a change in shape

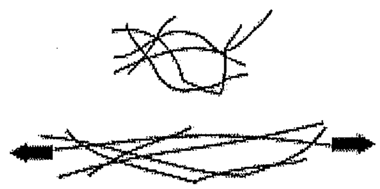


Elasticity

When materials are stretched, initially they will return to their original shape. As long as the stress applied to a material remains within its elastic limit, it will always return to its original shape. Elastic bands are of course the best example of elastic materials. They can be stretched a great deal without being permanently deformed. We call this *elastic deformation.*

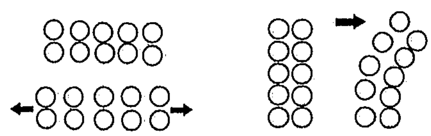
If stretched beyond its elastic limit a material will change shape permanently or *plastically deform.* Ductile and malleable materials can be deformed greatly while brittle materials will break.

Fig. 3a elastic bands are made of a tangle of chain-like molecules



They are so elastic because these molecules can straighten out when stretched without changing the actual structure or breaking

Fig. 3b elasticity in other materials



Elasticity in other materials relies on the bonds between molecules behaving a little like springs, but such materials are often not very elastic and will soon snap as the bonds break. Their strength depends upon the bond strength and the structure.

Stiffness
Stiffness is the resistance of a material to being bent, whether elastically or plastically. A steel sheet can be plastically (permanently) bent, a wooden longbow can be elastically bent, both require a considerable force and both are stiff.

Ductility
A ductile material is one which is relatively easy to stretch beyond the elastic limit. This is plastic deformation which means that once stretched, the material will remain that shape. If you take some Blu Tack© and carefully stretch it, it gets longer and thinner without breaking and retains its shape when the load is removed.

Copper is a ductile metal, although it does take a great deal of force to stretch it. Its ductility is one reason why it can be used in wiring as it can be stretched into long thin wires without breaking.

Fig. 4 a ductile material can be stretched permanently without breaking

Plastic deformation at the molecular level
In order for plastic deformation to occur without a substance breaking, there must be a change in its structure at a molecular level

Fig. 5 layers of atoms sliding over each other resulting in a permanent change of shape

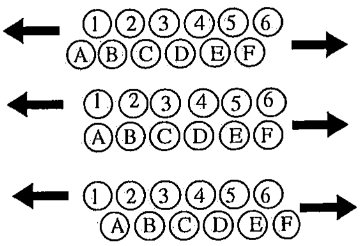


Fig 5 shows that a force can cause particles to slide over each other. No bonds are broken so the material does not break, but it can be permanently restructured into a new shape. Materials which can plastically deform (like ductile or malleable materials) can do this.

Brittleness and Malleability
When a material reaches its elastic limit, it can plastically deform if the load is increased. A brittle material will snap soon after it reaches this point. Often such materials are also not very elastic to begin with and so are fairly inflexible.

Glass is a common brittle material. Note that it is not the fact that glass is inflexible that makes it brittle; that simply means it is inelastic. Glass is brittle because once it has been bent elastically, it cannot be bent further in such a way that it permanently stays bent, it would simply break.

A substance that is malleable can be bent far beyond its elastic limit and will retain its new shape; they can be bent, rolled into sheets, pressed and hammered into shape. Where brittle materials would break, malleable ones retain their new shape without breaking.

Softness, Hardness and Toughness
The **harder** a material is, the more difficult it is to cut into, grind down, dent or mark in any way. Diamond, for example, is at the top of the hardness scale. Hard materials are often also brittle. For example glass is fairly inflexible, will snap easily but is very hard to scratch, cut or mark; as such it is brittle but hard. One of the only ways to cut or mark glass is to use diamond which is even harder.

A **soft** material can be very easily deformed (bent or misshapen), cut and marked. Blu Tack © is soft in that it takes very little force to change its shape. It is also very easy to cut and mark.

Note that ductile materials are not necessarily soft. Copper for example, is ductile as it can be permanently stretched without breaking, but this requires a great deal of force which means it is not soft.

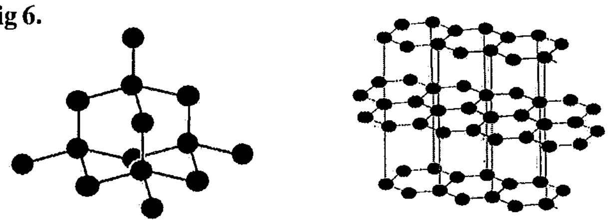
Soft materials are quite often also malleable, although malleable materials are not necessarily soft. It can take a great deal of force to press or roll some metals which means they are not soft. The properties that make Blu Tack © soft also make it malleable, steel is malleable but would not be described as a soft metal.

A **tough** material is a balance between soft and hard and as such is quite durable. Such materials can support loads without deforming easily, but are not so hard as to be brittle. Therefore when loaded, tough materials will 'give' a little in order that they do not break, but not so much that they completely bend and collapse. Also by being slightly flexible but hard, they can absorb impacts without shattering or deforming. A tough material must have a lot of work done on it before it breaks.

Strength
The strength of a material refers to how much force is required to break it; the stronger the material, the more force is needed. This amount of force per unit cross-sectional area is referred to as the breaking stress.

A material is broken when the atoms that make it up are separated. Therefore the strength of a material depends on the strength of the bonds between the atoms and the structure of the atoms.

Fig 6.



Diamond is so strong and hard thanks to the strong crystal structure of the carbon atoms.

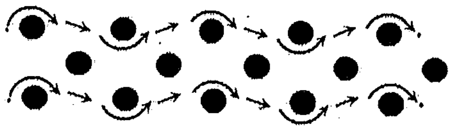
The carbon atoms in graphite are arranged in layers and the bonds between the layers are weak. Therefore graphite is not strong or hard as it can easily be broken and marked.

> ***Example 2:***
> ***Explain why diamond is useful to use to make the tip of heavy duty cutters, like glass or concrete cutters, whereas graphite would be useless, despite both being made from carbon.***
>
> *Answer*
> *A diamond tipped blade will not go blunt quickly and so can be used to cut dense materials like concrete without wearing out. This is because it is very hard and therefore difficult to mark, wear down or break. It takes a great deal of force to break any of the carbon atoms out of the strong crystal structure. In graphite the carbon atoms are arranged differently. They are arranged mainly in layers and there are very weak bonds between the layers. This means that it requires little force to break layers of graphite away from the main structure. This results in graphite being brittle and not very strong or hard. It can easily be marked or broken and so would wear down or break completely very quickly if used as a cutting tool.*

Electrical Conductivity and Resistance

Current is a flow of charge, so in order to conduct electricity a material must contain charged particles that can move. The more easily the charges can move and the more charges that are available, the greater the conductivity. Conversely the difficulty in getting charge to flow is a measure of electrical resistance. Resistance is measured in Ohms, Ω.

Fig 7



The movement of electrons in a metal as they move from ion to ion (atom to atom)

In solids it is electrons which carry this charge. They are loosely attached to the atoms and can move from orbiting one metal ion to the next and slowly work their way through the metal when a potential difference is applied. If all these loose (or 'free') electrons do this simultaneously we get a net flow of charge - which is current.

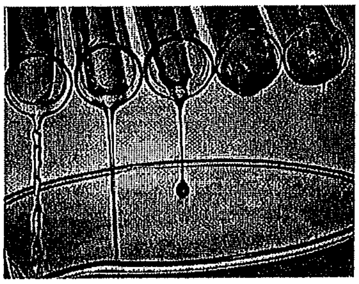
Generally metals have the highest conductivity though some non-metals such as graphite have free electrons and can therefore conduct electricity.

Viscosity

Viscosity is a property of liquids (although gases can be considered to have a viscosity too). The more viscous a liquid, the more slowly it will flow.

Syrup is more viscous than water as water flows faster. Viscosity is often dependent on temperature as viscous liquids flow more easily (become less viscous) in warmer conditions.

Fig. 8 different oils flowing



http://www.bestsynthetic.com/graphics/viscosity_3.jpg

They become more viscous as you move to the right of the picture. Image from

> ***Example 3:***
> ***Suggest why motor oil is not as effective as a lubricant when an engine has only just been started.***
>
> ***Answer***
> *When the engine is started the oil will be cold and so will be less viscous. It would therefore take longer to reach and lubricate all parts of the engine.*

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Practice Questions

1 The Earth has a radius of 6,500,000 m, and mass 6×10^{24} Kg.
(a) estimate the average density of the Earth. [1 mark]
(b) explain why this value is only an average. [2 marks]

2 Both aluminium and steel can be used to make a bicycle frame.
(a) State any properties which they both have which make them able to do this job and explain how these properties are useful. [4 marks]
(b) Why is aluminium a more common choice as a bicycle frame? [1 mark] (you may need to look up some properties of aluminium and steel to decide why aluminium is used)

3 Consider three objects that make use of one or more of the properties in the list below.
For each object, state the property and explain how it is being used. You can look at more than one property per object or material.

ductility	hardness	softness	high melting point
toughness	strength,	malleability	electrical conductivity
stiffness	brittleness point	low melting capacity.	high specific heat

Answers

1 (a) $\rho = \frac{m}{V} = \frac{m}{4/3\pi r^3} = \frac{6\times10^{24}}{4/3\pi 6500000^3} = 5216\ \text{kgm}^{-3}$ ✓
(b) The Earth is made up of many different materials and each will have a different density.✓
The density will also change with pressure as you go deeper below the Earth's surface.✓

2 (a) Similarities between aluminium and steel:
Tough ✓ – frame will not deform much under any normal amount of force and will not break if it is slightly deformed (i.e is not brittle) ✓
Strong ✓ - frame can withstand heavy loads without breaking ✓
Stiff – enables the frame to keep its shape
You could argue other properties but these are the main ones. A maximum of four marks, two for naming two properties and two for explaining how they are useful.
(b) Aluminium has a much lower density than steel and therefore would make the bike lighter while still being the same size and similar in strength and toughness.

3 One mark is awarded for naming the object, material and first property, up to three marks for three objects. Further marks are awarded for explaining how the named properties are used.
You must identify the material before the mark for usage can be awarded. Maximum of 6 marks. Some good answers would be:

Copper wiring makes use of ductility ✓ where the copper is stretched (drawn out) into correct shape for the wiring. ✓ Copper is also electrically conductive (low resistance and so easily carries electrical current) ✓

A metal (usually steel) chair leg is strong ✓which means it can withstand considerable loads without breaking. It is tough which gives it some flexibility to withstand sudden changes in load without shattering (it is very slightly flexible meaning it is more tough than hard. A hard metal could withstand high loads, but a tough material has some flexibility which gives an almost shock absorbing capacity, where a hard and brittle material would simply shatter)✓. It is stiff which means it will not easily bend when loaded. ✓All this means it can easily support not only a person's weight but also the effect of them sitting and moving around on the chair. Steel has a high melting point so that it remains a solid at all possible room temperatures.✓

Safety glass (like car side windows) is hard so that it cannot be easily damaged or marked in day to day use. ✓ It can withstand wear and tear without leaving getting easily scratched, which is important so that you can still see through clearly. ✓ It is also brittle so when struck hard it will shatter completely so that in the event of an accident the emergency services can gain access.✓ It also breaks into small squares to minimise sharp edges.

Physics Factsheet

www.curriculum-press.co.uk Number 147

Stress and Strain in Metals

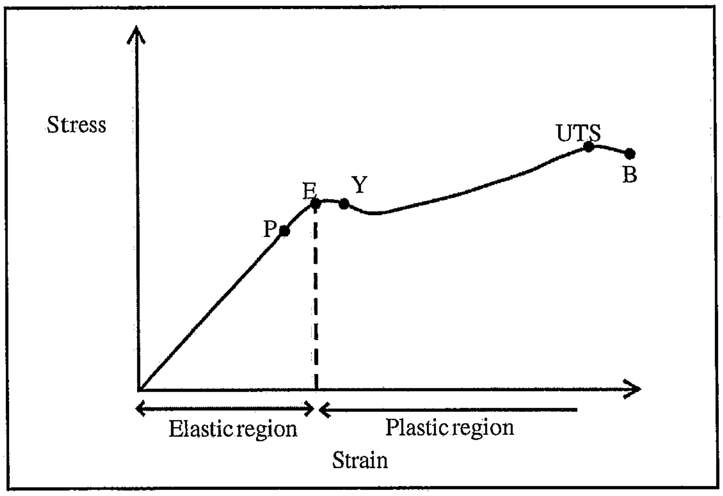
The greatest proportion of time spent studying the extension of metals (when a force is applied) is concerned with the metal in the elastic region. And we tend to specifically concentrate on the part of this region below the limit of proportionality – we look at Hooke's Law, the spring constant, and theYoung modulus.

This Factsheet will look more generally at metals when they are being extended. We will look at:
- different types of metals
- atomic structure
- relevant calculations

and we will study the metals right up to their breaking point.

Stretching to destruction

As mentioned, we want to see what happens to a metal wire as it is stretched until it breaks. Here is a typical example:



The axes are "stress" and "strain".

Stress is defined as the force per unit cross-sectional area, $\sigma=F/A$ (Nm^{-2} or Pa).
Strain is defined as the extension per unit original length, $\varepsilon=e/l_0$ (no units).

> *Stress and strain are properties linked to a* *__material__*, *rather than to a specific sample of the material. The stress-strain graph should be the same for any thickness and length of wire made of the same material.*

The graph itself is best explained by defining the labelled points:

P is the limit of proportionality. The extension up to this point is elastic, and obeys Hooke's Law. The strain is proportional to the applied stress. If the stress is relaxed, the wire returns to its starting length.

E is the elastic limit. Up to this point, the wire will return to the starting length if the stress is relaxed. However the graph is curving – Hooke's Law is no longer obeyed.

> *Up to the elastic limit, E, the wire has not been permanently deformed. The implications for stored energy are discussed later in the factsheet.*

Y is the yield point. The wire is now being permanently deformed, and at point Y there is a sudden increase in strain actually accompanied by a reduction in applied stress.

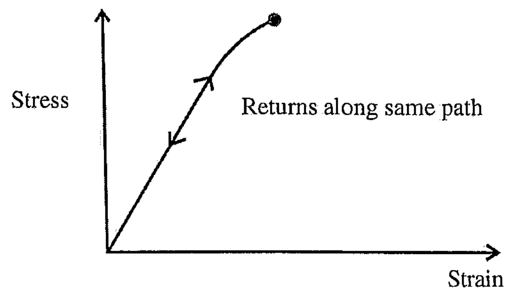
UTS is the ultimate tensile stress. This is not necessarily the breaking point. The wire is deforming plastically. The wire will now continue to stretch with reduced applied stress.

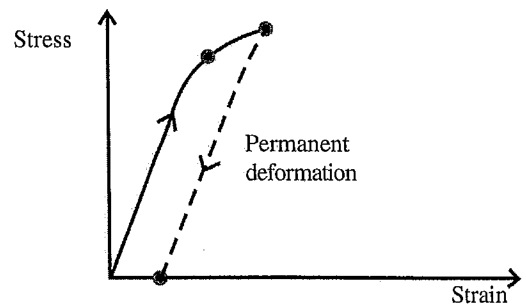
B is the breaking point. The wire breaks at a weak point. The value of the strain at this point is a property of the individual sample of wire.

> *After the elastic limit, the extension of the wire is in the plastic region. Permanent deformation is taking place. As we will see, there are implications for energy transfer.*

Example 1:
Sketch graphs showing what happens to the wire if the stress is relaxed when the wire is at its elastic limit, E, and when it is at its yield point, Y.

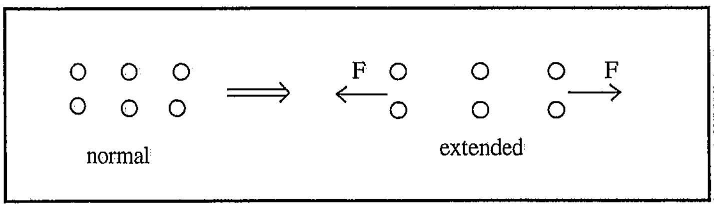
Answer:





Atomic structure explanation
Very briefly, we will look at what happens in the elastic and plastic regions.

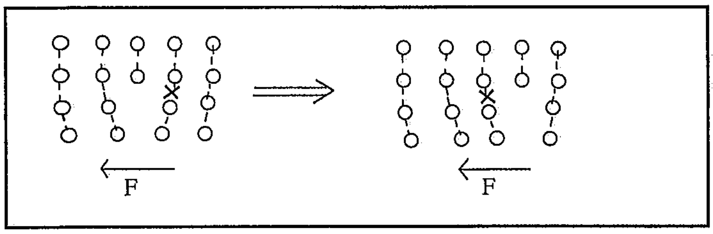
(a) Elastic



The atoms in the lattice stay in position relative to each other. However the applied force pulls them away from each other. At first the extension is proportional to the applied force. As the force is increased, the extension is no longer proportional. However the atoms still return to their original positions when the force is relaxed.

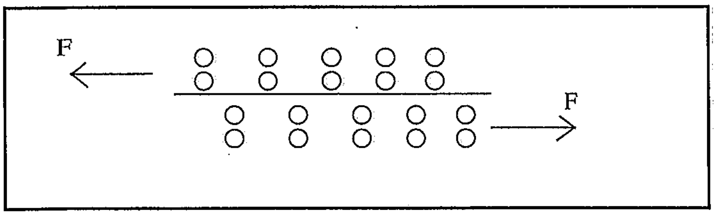
> *The energy transferred to the lattice as elastic potential energy can all be used to do work when the lattice returns to its original shape.*

(b) Plastic
In the plastic region a number of processes occur.



At a dislocation in the crystal structure, the applied force has broken a bond causing a crystal plane to move along one position. The sample has undergone permanent deformation.

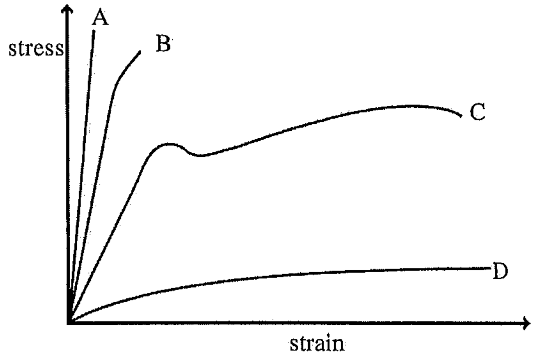
And if the applied force is increased even more, whole planes of atoms can slide past each other, greatly increasing the strain in the wire.



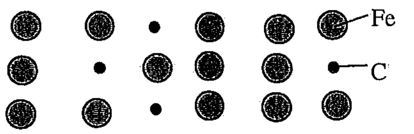
Again, the deformation is permanent.

> *The energy required to cause permanent deformation cannot be regained. It will have been transformed into thermal energy.*

Different types of materials
The original graph is representative of many metals. However different types of metals will give different responses when a force is applied.



A is a <u>brittle</u> material e.g. cast iron. It deforms elastically, but there is very little strain even for a large applied stress. Then it suddenly breaks. Cast iron has a high proportion of "impurity" atoms (carbon) introduced. The different atom sizes make plastic deformation very difficult.

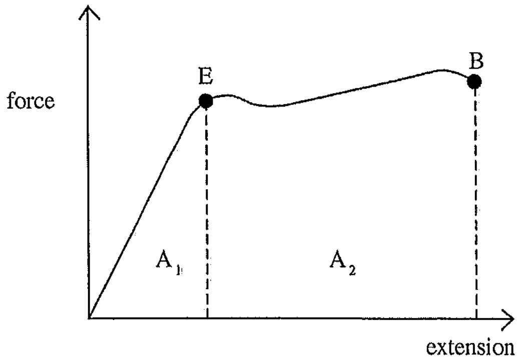


B is a <u>strong</u> material e.g. steel. There are fewer "impurity" atoms (carbon). More movement is possible, but it is still very difficult for plastic deformation to occur.

C is a <u>ductile</u> material e.g. pure copper. The lack of impurity atoms makes it much easier for atom planes to slide past each other. Most pure metals undergo plastic deformation.

D is a <u>plastic</u> material e.g. pure lead. It is so easy to slide the crystal planes past each other that there is very little elastic stretching.

Energy under a Force-Extension Graph
A force-extension graph is very similar in shape to a stress-strain graph. The area under the force-extension graph gives a value for the work required to stretch the wire sample.



This wire is stretched past its elastic limit, E, right through its breaking point, B.

Work done in breaking the wire, $W = A_1 + A_2$

Energy stored as elastic potential energy in the wire, $EPE = A_1$

Energy "lost" as thermal energy, $E = A_2$

Example 2:
Why is a steel wire much more dangerous when it breaks than a copper wire?

Answer:
Almost all of the work done on the steel wire has been stored as elastic potential energy (rather than transferred into thermal energy). The release of energy upon breaking can cause the steel wire to "whiplash" violently.

Example 3 :
Why is a force-extension graph slightly different in shape to a stress-strain graph?

Answer:
The strain will be proportional to the extension. However, as the wire stretches, its cross-sectional area decreases. As $\sigma = F/A$, the changing area will mean that the stress will not be exactly proportional to the applied force.

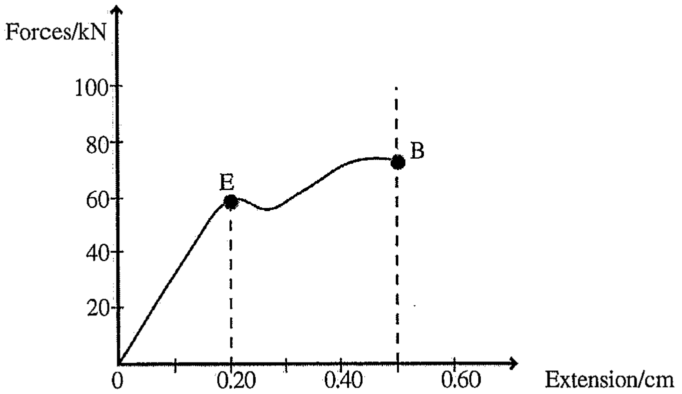
Calculations

The difficulties with calculations arise from the large values for force, combined with the small values for extension. Force may be measured in kN, and extension may be fractions of a millimetre. And the radius of the wires involved may also be less than a millimetre.

***Exam Hint:-** Take care with the detail of the calculations in this topic. Notice whether units are provided in N or kN (for example).*

Practice Questions

1. Estimate the elastic potential energy stored in this wire before it breaks, and the energy transferred to heat in stretching it.



2. Find the maximum load that can be supported by a steel cable 1.2cm in diameter. (Ultimate Tensile Stress = 500MPa)

3. A hammer thrower swings a hammer (mass 6.8kg) in a horizontal circle of radius 1.65m. The speed of the hammer is $6.5ms^{-1}$. The steel wire has a radius of 2.5mm.
   (a) Find the centripetal force provided by the tension in the wire.
   (b) Find the stress in the wire.

4. (a) Calculate the stress in a copper wire of radius 0.20mm when it supports a mass of 2.5kg.
   (b) Calculate the stress in the same wire if the load was doubled to 5.0kg.
   (c) What assumption have you made in calculating (b)?

5. Cast iron has a relatively high proportion of impurity atoms (carbon). It is brittle. Steel has fewer carbon atoms. It is more flexible, and is classed as a strong material. Wrought iron has very few carbon atoms in the lattice. What sort of material would you expect it to be?

Answers

1. $EPE = 0.5 \times (0.20\times10^{-2}) \times (60\times10^{3}) = 60J$
   $Heat = (0.3\times10^{-2}) \times (65\times10^{3}) = 195J$

2. $\sigma = F/A$, $F = \sigma\times A = (5.0\times10^{8}) \times \pi \times (0.6\times10^{-2})2 = 5.7\times10^{4}N$

3. (a) $F = \frac{mv^2}{r} = \frac{6.8\times6.5^2}{1.65} = 174N$

   (b) $\sigma = \frac{F}{A} = \frac{174}{\pi\times0.0025^2} = 8.9\times10^6\,Pa$

4. (a) $\sigma = \frac{F}{A} = \frac{2.5\times9.81}{\pi\times(0.20\times10^{-3})^2} = 2.0\times10^8\,Pa$

   (b) $4.0\times10^{8}Pa$

   (c) We are assuming that the cross-sectional area has not decreased significantly. This probably means that we are operating in the elastic region.

5. You might expect it to be ductile. Movement of crystal planes past each other should be much easier.

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Physics Factsheet

January 2002 Number 27

The Young Modulus

This Factsheet covers the quantity known as 'the Young modulus' which is used when describing how materials behave when they are stretched by a force. In order to understand the Young modulus we must first consider the quantities of stress and strain, which are used in defining the Young modulus of a material.

Stress

One important factor that affects how a material behaves when it is stretched by a force is the size of the force being used to stretch the material. A bigger force will lead to a bigger stretch. The effect of this stretching force (or *'tensile force'*) will be different for thick blocks and for thin wires of the same material.

Another important factor to consider when making comparisons between materials is the cross-sectional area of the material being used.

Tensile stress is the force being applied per unit cross sectional area (1 m^2) of the material.

> ***Tensile Stress is defined as:***
> *The force acting on unit cross sectional area of material.*
>
> $$\sigma = \frac{F}{A}$$
>
> σ = tensile stress (Pa)
> F = force (N)
> A = cross sectional area (m^2)

The symbol used for tensile stress is the Greek letter σ. Stress has the units Nm^{-2} which is exactly the same as pascals Pa; both units are commonly used.

So, when a force is stretching a material, physicists and engineers will quote the tensile stress that the material is under rather than the force being used, as the tensile stress takes account of the cross sectional area of the material being stretched.

> ***Exam Hint:*** *A common mistake in exam answers is to leave out the words 'cross sectional' when describing the area of the material. These words are important and should always be included in the definition.*

> ***Breaking Stress is defined as:***
> *The stress at which a material will break or 'snap'.*

Strain

The effect of applying a tensile stress to a material is a stretch, or an extension in length of the material.

The amount of this extension also depends on the original length of the materia; a longer piece of material will stretch further than a shorter piece even though they are under the same stress.

The **tensile strain** of a material is the extension per metre of the material.

> ***Tensile strain is defined as:***
> *The extension per unit length of the material.*
>
> $$\varepsilon = \frac{e}{l}$$
>
> ε = tensile strain (no units)
> e = extension (m)
> l = original length (m)

The symbol used for strain is the Greek letter ε. Strain has no units as it calculated by dividing one quantity of length by another quantity of length.

So, when a material is being stretched physicists and engineers will refer to the tensile strain of the material as this takes account of the length of the material.

> ***Exam Hint:*** *When exam questions ask for the definition of tensile stress or tensile strain, the equation can be used as well as the definitions given in words above, but don't forget to define the symbols used in the equations.*

> ***Breaking Strain is defined as:***
> *The strain at which a material will break or 'snap'.*

Young Modulus

> ***Young Modulus is defined by an equation as:***
>
> $$E = \frac{\sigma}{\varepsilon}$$
>
> E = Young modulus (Nm^{-2})
> σ = stress (Nm^{-2})
> ε = strain

The Young modulus has the units of Nm^{-2}, or Pa, the same units as stress. The Young modulus is a measure of the 'stiffness' of the material. A large Young modulus indicates that a large stress is required to produce a small strain and the material does not deform easily. Engineers use the Young modulus for different materials as it does not depend on the dimensions of the material that they are using. It is a quantity for each material that can be used regardless of its shape and size.

Using the equations that have already been introduced for stress and strain, we can substitute into our equation for Young modulus:

> $$E = \frac{\sigma}{\varepsilon} = \frac{(F/A)}{(e/l)} = \frac{Fl}{Ae}$$
>
> F = force (N)
> l = original length (m)
> A = cross sectional area (m^2)
> e = extension (m)

This gives us an expression for Young modulus using quantities that can be directly measured from the material being stretched.

Exam Hint: *Don't be put off if your values for Young modulus seem large. Typical values for Young modulus vary from rubber, Young modulus = 7×10^5 Pa, to steel, Young modulus = 2×10^{11} Pa.*

Typical Exam Question
As part of a quality control check, a manufacturer of washing line subjects a sample to a tensile test. The sample of washing line is 12m long and of constant circular cross section of diameter 5.0mm. The manufacturer measures an extension of 42mm under a stretching load of 72N.
The manufacturer also breaks the line under a load of 240N.

(a) Calculate the Young modulus of the washing line. [3]

(b) Calculate the breaking stress of the line. [2]

(c) If the Young modulus of the line stays constant throughout what is the extension of the line just as it breaks? [3]

Answer
(a) The Young modulus equation requires the cross sectional area of the material so we must first calculate the cross sectional area of the washing line. The cross section of the line is circular and care must be taken as the question gives a measurement for **diamete**r in **millimetres.** The measurement must be in metres when used in the equation and the usual equation for areas of a circle requires the radius.

$A = \pi r^2 = \pi(0.005/2)^2 = 1.96 \times 10^{-5} m^2$. ✓

We now have all the figures to substitute into our equation for Young modulus, remembering to change my extension into metres.

$E = Fl/Ae = (72)(12)/(1.96 \times 10^{-5})(0.042) = 1.05 \times 10^9 Pa$ ✓ ✓

(b) When calculating the breaking stress we must use the value of force at which the line breaks. We already have a value for the cross sectional area of the line from part (a) of the question.

$\sigma = F/A = 240/1.96 \times 10^{-5} = 1.22 \times 10^7 Pa$ ✓ ✓

(c) *This part of the question is most easily understood if it is completed in two stages. Firstly, knowing that the Young modulus remains unchanged as well as the breaking stress, the breaking strain can be calculated.*

$E = \sigma/\varepsilon$

Rearranging this equation, strain can be made the subject of the equation

$\varepsilon = \sigma/E = 1.22 \times 10^7/1.05 \times 10^9 = 0.0117$ ✓

Knowing the breaking strain and the original length the extension can now be calculated. ✓

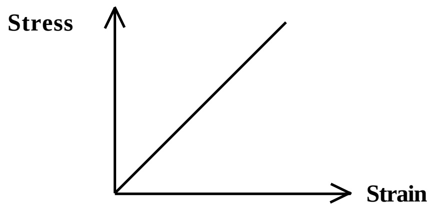
$\varepsilon = e/l$

Rearranging this equation, extension can be made the subject of the equation.

$e = \varepsilon\, l = (0.0117)(12) = 0.14m$ ✓

Stress – Strain Graphs

For a great many materials, an increase in stress leads to an increase in strain i.e. a bigger stretching force produces a bigger extension. For materials such as metals and glass, if the stress is doubled, then the strain will be doubled also. A graph of stress on the vertical axis against strain on the horizontal axis would therefore be a straight line through the origin.



This graph shows that stress increases at the same rate as strain, or stress is directly proportional to strain.

Hooke's Law
*A material that has stress proportional to strain is said to **obey Hooke's Law.** A material that obeys Hooke's law undergoes elastic deformation. This means that when a force is no longer applied to the material it will return to its original shape and will not have any permanent stretch or extension.*

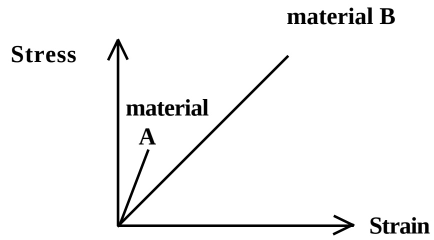
The gradient of this graph remains constant and can be calculated by dividing the change in *y* by the change in *x*. On the graph you are effectively dividing a stress by a strain. This is the same as calculating the Young modulus of the material.

Calculating Young modulus from a stress – strain graph
On a stress–strain graph the Young modulus is given by the gradient

$$\text{Young modulus} = \frac{stress}{strain} = gradient$$

The stress – strain graph below has two lines on it for two different materials, A and B. Both of the lines are straight lines through the origin, showing that both materials obey Hooke's law.
- The line for material A has a steeper gradient than the line for material B showing material A to have a larger Young modulus; material A is '**stiffer**' than material B.
- Note also how the line for material B finishes higher than material A. This shows that material B has a larger breaking stress than material A; material B is '**stronger**' than material A (Fig 1).

Fig 1



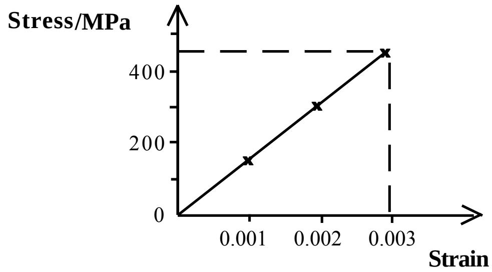
Typical Exam Question

The graph below is the stress-strain graph for a metal that obeys Hooke's law over the region covered by the graph.

(a) What is the Young modulus of the metal? [3]

A wire made from this metal has a diameter of 0.15mm and a length of 2.5m.

(b) Calculate the extension of the wire under stress of 200MPa. [2]



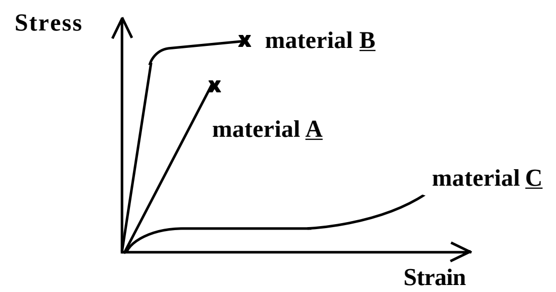
The Young modulus is simply given by the gradient of the graph. Remember that the vertical axis is in MPa.

(a) Young modulus, E = gradient of graph
= change in height/change in length ✓
= $450 \times 10^6/0.003 = 1.5 \times 10^{11}$Pa. ✓

(b) Reading from the graph, the metal has a strain of 0.0010 when under a stress of 150MPa. ✓
This strain can now be used in the equation for strain:
extension, e = εl = (0.001)(2.5)= 2.5×10^{-3}m ✓

Not all stress-strain graphs are straight lines through the origin. Stress-strain graphs can also be curved or a combination of a straight line followed by a curve. The graph below shows three different materials (Fig 3).

Fig 3



Material **A** gives a straight line right up until the material breaks, represented by a cross on the line. This material obeys Hooke's law and breaks very suddenly. The material is brittle. An example of this material is **glass** Material **B** also obeys Hooke's law initially. It is also stiffer than material **A** as it has a larger Young Modulus represented by the steeper gradient. The Young modulus of material **B** decreases rapidly beyond the point where Hooke's law is obeyed. In this region the material gives a larger strain for increases is stress, this means it deforms easily and is ductile. An example of this material is most **metals**.

Material **C** does not have a constant Young modulus and it is more ductile than material **B**. It also stretches much more before it breaks. An example of this material is rubber.

A brittle material does not behave plastically, it does not change its shape before breaking. A ductile material behaves plastically and will easily change its shape before it breaks.

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The Examiner's answer is given below.

(a) When determining the Young modulus of a wire, a tensile stress is applied to the wire and the tensile strain produced is measured.

(i) Define the term: 'tensile stress' [2]
stress = force/area ✓ 1/2

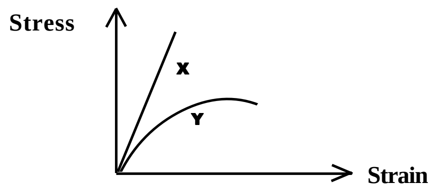
It is important that cross sectional area is specified.

(ii) Define the term: 'tensile strain' [1]
Strain = extension/length ✓ 1/1

Although this would probably get the mark, original length should be specified. It is also good practice to define both of the above terms in words as well as equations to leave no doubt in the examiner's mind that you should have the marks.

(iii) Define Young modulus [1]
Young modulus = stress/strain ✓ 1/1

(b) The graph represents stress-strain curves for two different materials X and Y. Both materials are stretched until they break.



(i) Define the term: 'ductile' [2]
Bendy 0/2

This word is not very precise or scientific!

(ii) Which of the materials is ductile? [1]
Material Y ✓ 1/2

(iii) State and explain which of the two material has a greater value of Young modulus. [2]
Material X as it is steeper ✓ 1/2

Although the student has identified the correct material the explanation is difficult to understand and poorly explained.

(c) A fishing line of length 3.0m and diameter 0.20mm supports a load of 25N. Given that the Young modulus for the line is 1.7×10^{10}Pa, calculate the extension in the line produced by this load. [3]
Area = $\pi r^2 = \pi(0.2)^2 = 0.13$
E = Fl/Ae, e=Fl/AE = (25)(3)/(0.13)(1.7 × 10^{10}) = 3.4×10^{-8}m ✓ 1/3

The diameter instead of the radius has been used in the calculation of area. Millimetres have not been changed into metres when calculating area.

Examiner's Answers

(a) (i) The force acting on unit cross sectional area of material.
Tensile stress = tensile force/cross sectional area. ✓ ✓
(ii) The extension per unit length of the material.
Tensile strain = extension/original length ✓
(iii) Young modulus = stress/strain ✓

(b) (i) A ductile material behaves plastically and changes its shape before it breaks. ✓ ✓
(ii) Material Y ✓
(iii) Material X has the greatest Young modulus, because the gradient of the stress-strain curve, which represents Young modulus is steepest. ✓ ✓

(c) A = $\pi r^2 = \pi(0.1 \times 10^{-3})^2 = 3.14 \times 10^{-8}$m^2. ✓
E =Fl/Ae = (25)(3)/(3.14 × 10^{-8})(1.7 × 10^{10}) = 0.14m ✓ ✓

Qualitative Test

1. What is breaking stress?

2. Why do engineers prefer to use quantities such a s stress and strain rather than force and extension?

3. How will a strong material differ from a weaker material?

4. How are stress and strain related for a material that obeys Hooke's law? How is this shown on a stress-strain graph?

5. Define a brittle material.

6. How will the stress strain curve for a strong material differ from that of a weaker material. Explain your answer.

7. Describe the stress strain graph of a ductile material.

Quantitative Test

1. A material of diameter 2.0mm supports a load of 60N. Calculate the stress exerted on the material.

2. A material has a breaking strain of 0.075. If a wire has an original length of 0.50m calculate the extension of the material as it breaks.

3. A wire of length 1.5m and diameter 0.25mm extends by 1.5mm when a load of 120N is placed on the wire.

   (a) Calculate the Young modulus of the wire
   (b) What extension would a load of 85N produce?
   (c) What is the strain of the wire with a load of 85N?

4. When certain rocket engines are fired they produce a total thrust force of 4.2×10^6N. In test firing, the rocket is held to the launch pad by 6 steel bolts, each of diameter 8.0 cm. The Young modulus of steel is 2.0×10^{11}Pa. The breaking stress of steel is 5.0×10^8Pa

   (a) Calculate the strain for each bolt during the test.
   (b) Determine the minimum number of bolts that could have been used for testing the engines.

Quantitative Test Answers

1. $A = =\pi r^2 = \pi(1 \times 10^{-3})^2 = 3.14 \times 10^{-6} m^2$ ✓ ✓
   $\sigma = F/A = 60/(3.14 \times 10^{-6}) = 1.9 \times 10^7 Pa$ ✓

2. $e = \varepsilon l = (0.075)(0.5) = 0.0375m$ ✓ ✓

3. (a) $A = \pi r^2 = \pi\ (0.125 \times 10^{-3})^2 = 4.9 \times 10^{-8} m^2$ ✓
   $E = Fl/Ae = (120)(1.5)/(4.9 \times 10^{-8})(0.0015) = 2.4 \times 10^{12} Pa$ ✓ ✓

   (b) $e = Fl/AE=(85)(1.5)/4.9 \times 10^{-8})(2.4 \times 10^{12}) = 1.1 \times 10^{-3} m$ ✓ ✓

   (c) $\varepsilon = e/l = 1.1 \times 10^{-3}/1.5 = 7.3 \times 10^{-4}$ ✓ ✓

4. (a) Area of 1 bolt $= \pi r^2 = \pi(0.04)^2 = 5.03 \times 10^{-3} m^2$ ✓ ✓
   $\sigma = F/A = 4.2 \times 10^6/(6 \times 0.005) = 1.4 \times 10^8 Pa$ ✓ ✓ ✓

   (b) min number of bolts = total strain/breaking strain per bolt
   $= 5 \times 10^8/1.4 \times 10^8 = 3.57 = 4$ bolts ✓ ✓

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Physics Factsheet

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Experiments- Hooke's Law and Young Modulus

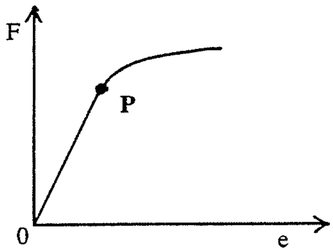
Factsheet 27 went through the elasticity theory required at A-level (and probably further) in some detail.

In this Factsheet we will look at some of the experimental work linked to the topic. We will concentrate on how best to make the practical work produce accurate and reliable data, and the graphical work and calculations resulting.

Hooke's Law provides information on the properties of a **specific device** (spring, length of wire, etc). The **Young modulus** gives us a value for a **material** (steel, copper, glass, etc).

> Hooke's Law refers to a specific device; the Young modulus refers to a material.

Hooke's Law (revision):

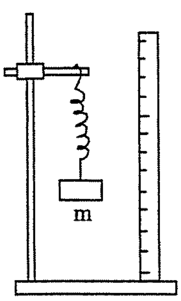


In the proportional region, between O and P (the **limit of proportionality**):

> $F = ke$ *where* F *is the applied force (N)*
> e *is the extension (m)*
> k *is the spring constant (Nm^{-1})*

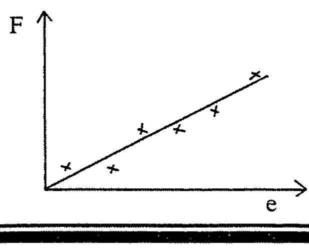
Practical Hint:
Throughout these practicals, always do repeats and averages where possible, and take care with significant figures and units.

Finding the spring constant, k, of a steel spring.



This is a standard practical going back to GCSE level. A series of masses are carefully added to the mass holder, and measurements of extension and weight are recorded in a table.

The results are then graphed:



The spring constant, **k**, is the gradient (from $k = \Delta F / \Delta e$).

Practical Hints:
1. *Use only small masses. This gives you more data points. It also makes it less likely you will exceed the limit of proportionality.*
2. *Repeat readings with decreasing masses to ensure there is no hysteresis effect.*
3. *Some springs are manufactured with the coils forced so tightly together that it takes a significant force to begin separating them. This may affect the starting point of the graph. Use only the straight-line section to find the gradient.*

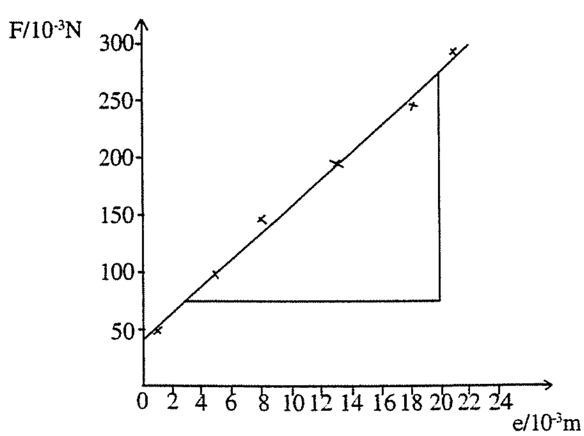
Example:
A student performs an experiment to find the spring constant of a steel spring, obtaining these results:

Mass / g	Ave. extension / 10^{-3} cm
0.0	0.0
5.0	0.1
10.0	0.5
15.0	0.8
20.0	1.3
25.0	1.8
30.0	2.1

Find the spring constant.

Solution:
Rewrite the table:

Weight/10^{-3}N	Ave. extension / 10^{-3} m
0	0
49	1
98	5
147	8
196	13
245	18
294	21

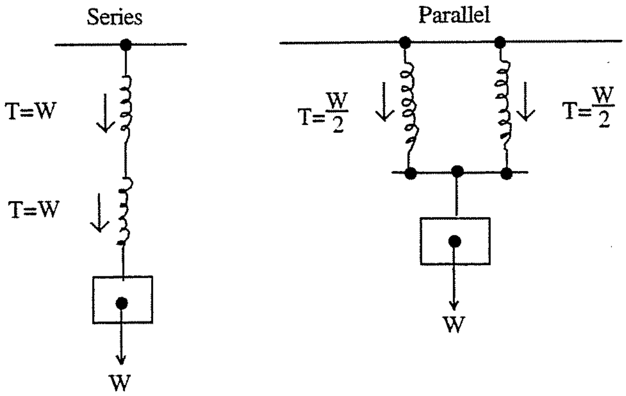


k = gradient = $(200 \times 10^{-3}) / (17 \times 10^{-3}) = 12\ Nm^{-1}$

Notice the care that must be taken with the units, and that the best straight line should not be started from the origin in this situation.

Combinations of Springs

Experimental work is often performed to verify the rules for combining springs in series and parallel.



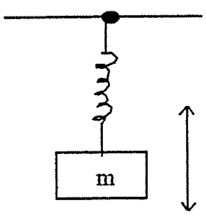
Practical Hints:
In the series combination, different springs can be used, but be prepared to add the inverse spring constants:

$$1/k = 1/k_1 + 1/k_2$$

*The most common error is forgetting to invert **1/k** to find **k** at the end of the calculation.*

*In the parallel combination, **always** use identical springs. Otherwise one will extend further than the other, causing the lower support rod to tip.*

Finding "g" from SHM with an oscillating steel spring:



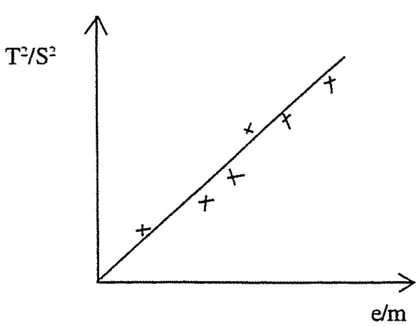
Hooke's Law can be used in simple harmonic motion where the period of the mass, m, oscillating vertically from a spring depends on the spring constant.

$T = 2\pi\sqrt{(m/k)}$

Using $k = \frac{F}{e} = \frac{mg}{e}$

We find $T = 2\pi\sqrt{(e/g)}$

Using a range of masses, we record extension e and period of oscillation **T**.



The gradient of the graph will be $4\pi^2/g$.

Practical Hints:
Use small extensions for the spring (small masses) to ensure that you are operating in the Hooke's Law region. However remember using very small extensions will increase the percentage error in the measurement. Use small amplitude oscillations to stay within the Hooke's Law region, and also to reduce the likelihood of the spring entering "swinging mode" (acting like a pendulum).

Example:
With the previous set-up, the student sets the mass on the steel spring into vertical oscillation. The time for 10 oscillations is measured (and repeated and averaged) for each mass.

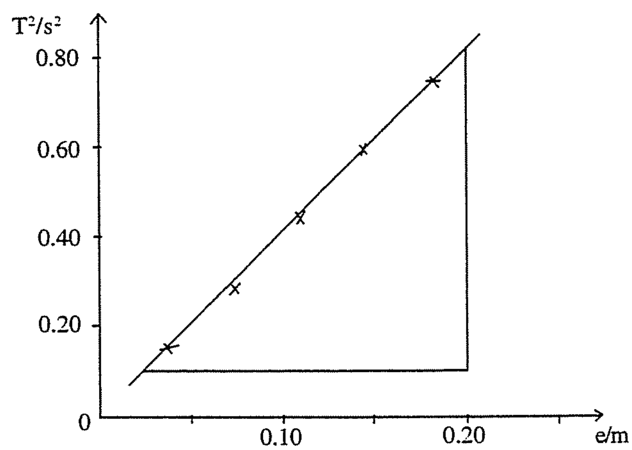
Table of results:

Mass / g	Ave. time for 10 osc. / s	Extension, e / cm
50	3.8	3.7
100	5.4	7.3
150	6.7	11.1
200	7.7	14.9
250	8.6	18.5

Find the value of "g" from the results.

Solution:

Period², T^2 / s^2	Extension, e / m
0.144	0.037
0.292	0.073
0.444	0.111
0.596	0.144
0.740	0.185



The gradient works out to be 4.0 s^2m^{-1}.

$4\pi^2 / g = 4.0$

$g = 9.87\ ms^{-2}$.

Practical Hints:
Dynamic measurements (e.g. timing oscillations) are more difficult than static measurements (e.g. measuring extension). It is essential to repeat and average, and to measure ten or twenty oscillations, not just one.

The Young modulus (revision):
As mentioned, the Young modulus is a property of a material. Instead of applied force, we use **tensile stress** (the force applied per unit cross-sectional area); instead of extension, we use **tensile strain** (the fractional increase in length).

> *Stress = F/A* (Nm^{-2})
> *Strain = e/l* *(no units) where l is the original length.*
>
> *Young modulus, E = stress/strain = (F/A) / (e/l)*
>
> *Or we can write: E = (Fl) / (Ae)* (Nm^{-2})

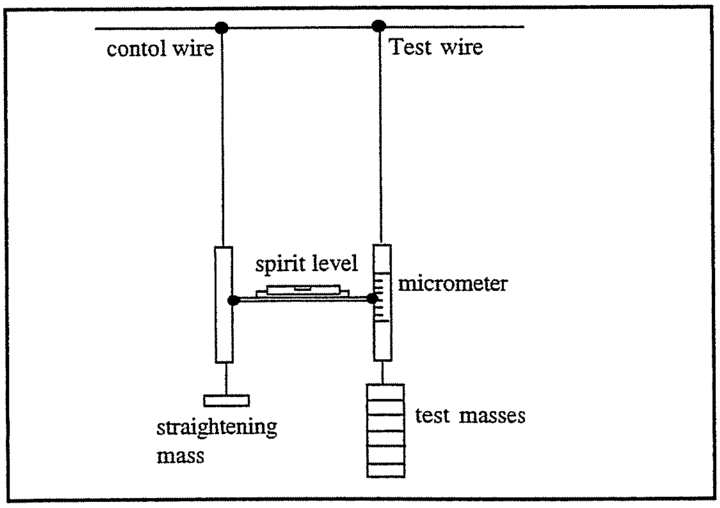
> The Young modulus is a measure of the stiffness of a material

It may be useful to emphasise the size of the effect we may see experimentally with an example.

Example:
Mild steel has a Young modulus $E = 20 \times 10^{10}$ Nm^{-2}. A steel wire of length 1.00m has a cross-sectional area of $1.0mm^2$. Find the extension if a force of 100N is applied to the wire.

Solution:
$E = (Fl) / (Ae) = (100\times1.0) / (1.0\times10^{-6} \times 20\times10^{10}) = 5.0\times10^{-4}m = 0.50mm.$

The point of this example is to illustrate the tiny extension expected in Young modulus investigations.

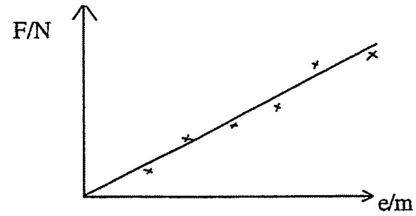


This set-up, combined with careful experimentation, should maximise the accuracy of the final result. **Notice these points – they are important:**

1. A micrometer is used to measure the diameter in several places on both wires.
2. The control wire compensates for changes in temperature or "sagging" of the support frame.
3. Small loads are introduced onto both wires to make sure they are straight before measurements are taken.
4. The spirit level and micrometer allow very small changes to be measured.
5. Readings on both loading and unloading are taken for accuracy, and to check for any hysteresis effect.
6. A best-straight line graph is used for increased accuracy.

> **Safety point:** To maximise the extension obtained, a long and very thin wire is put under considerable tension. Goggles must be worn in case the wire snaps.

A graph of results will resemble this example:



E = (Fl) / (Ae)

So the experimental value for E will be found from:

E = gradient × **(l/A)**

> ***Exam Hint:*** *Be prepared to discuss the ways in which the extension of the wire is maximised, and the steps taken to improve accuracy. Also be ready to discuss why accuracy is more difficult to achieve in finding the Young modulus than the spring constant.*

Questions

1. (a) State the key difference between the spring constant and the Young modulus.
   (b) Why is the spring constant generally easier to determine experimentally?
2. Using weights from 0 to 10N, a steel spring exhibits extensions increasing linearly from 0 to 20cm.
   (a) Find the spring constant.
   (b) Find the extension for a load of 25N. (Can you be certain of this?)
3. Here is a set of results for an experiment with a length of metallic wire (original length, l = 1.5m ; diameter = 2.0mm).

Tension, T / N	Extension, e / mm
0	0.00
100	0.19
200	0.42
300	0.60
400	0.83
500	0.99
600	1.24

   (a) Graph these results (with **T** on the y-axis).
   (b) Find the gradient (in Nm^{-1}).
   (c) Find the cross-sectional area (in m^2).
   (d) Calculate the Young modulus for this metal.
4. A 1.0m length of wire is stretched by 0.5mm. This causes a decrease in the cross-sectional area.
   (a) Estimate the % change in the stress due to this area change.
   (b) Is this large enough to be noticeable in the results?

Answers:

1. (a) Spring constant – property of device
   Young modulus – property of material.
   (b) Much larger extension when dealing with springs, etc.
   Easier to measure extension accurately.
2. (a) **$k = F/e = 50Nm^{-1}$**
   (b) 50cm (but only if the limit of proportionality is not exceeded)
3. (b) Gradient approximately $4.9 \times 10^5 Nm^{-1}$.
   (c) $3.14 \times 10^{-6} m^2$
   (d) **E = gradient × $(l/A) = 23 \times 10^{10} Nm^{-2}$**
4. (a) **stress = F/A, volume = Al**
   % change in stress of same magnitude as that in area.
   And % change in area of same magnitude as that in length.
   % change = 0.05%
   (b) This is too small to be noticed.

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